Nonlinear observer-based control for PMSG wind turbine

Roberto Fantino, Jorge Solsona*, Claudio Busada

Instituto de Investigaciones en Ingeniería Eléctrica (IIIE) "Alfredo Desages" (UNS-CONICET), Departamento de Ingeniería Eléctrica y de Computadoras, Universidad Nacional del Sur (UNS), 8000, Bahía Blanca, Argentina

ARTICLE INFO
Article history:
Received 11 December 2015
Received in revised form 30 June 2016
Accepted 7 July 2016
Available online 19 July 2016

Keywords:
Permanent magnet synchronous generator
WECS
Sensorless control
Nonlinear observer
MPPT

ABSTRACT
This paper introduces a mechanical sensorless control strategy to be used in variable speed wind energy conversion systems based on permanent magnet synchronous generators. The goal of the control strategy is to track the wind turbine maximum power point. With the proposed strategy, mechanical sensors are not needed to implement the control law. For this reason, electrical sensors—voltage and current sensors—are only needed to build the control law.

In order to obtain estimates of the mechanical variables, a nonlinear Luenberger-like observer is designed. This observer only uses measurements of the electrical variables. The estimates are fed back to the controller in order to build the mechanical sensorless control strategy. The performance of the whole system is tested through simulations.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

Wind energy conversion systems (WECS) used in distributed generation must be able to inject energy in an efficient way. To this end, nowadays, the use of speed variable permanent magnet synchronous generator (PMSG) has increased a lot. It is mainly because PMSGs based WECSs have high efficiency and they reduce operation and maintenance cost, since this scheme does not employ neither gearbox nor slip rings. In addition, PMSGs based WECSs can be controlled for wind turbine maximum power point transfer (MPPT) operation, in a wide range of the wind speeds [1,2].

In order to transfer power from the generator to the grid, most of WECSs include Voltage Source Converters (VSCs). However, over the last few years, the use of topologies including Current Source Converters (CSCs) have increased a lot. This is because these topologies present several advantages [3,4]: 1) natural protection against shortcircuit 2) low dv/dt variations of the output voltage 3) high reliability of conversion and 4) regenerative capacity [5]. There are different configurations for WECS based on CSCs. One of the best, when it comes to power quality, consists of a current source rectifier (CSR) and a current source inverter (CSI) connected in a back-to-back configuration [6–8].

Many times, when the turbine power coefficient \(C_p(\lambda, \beta)\) is known, the power can be controlled to track the maximum power point. To this end, control algorithms for MPPT operation are implemented (see for instance, [9–12] and references therein). In such a case, the PMSG torque must be controlled [13–16] by setting the torque reference as a function of the mechanical variables. It is possible to measure mechanical variables to implement the control strategy. Nevertheless, in order to reduce cost and to increase the system reliability, sensorless algorithms can be used to estimate them. It can be mentioned that in Ref. [17], many of the problems that must be solved in order to obtain a more reliable and smart wind energy system were identified. There, optimum control strategies for PMSG wind turbine systems without mechanical sensors are mentioned. Some researchers have already proposed mechanical sensorless strategies. For instance, in Refs. [15,18] a PLL is used to estimate the turbine speed and the rotor speed. This strategy does not consider the acceleration in its prediction model. However, including the acceleration in the prediction model improves the estimator transient performance, and therefore, it is included in the proposal of this manuscript. In Ref. [19] a low-pass filter is used to estimate the stator flux, and then the mechanical speed is estimated via a recursive LMS algorithm. In Ref. [14] the stator flux and the rotor mechanical speed are estimated with a quasi sliding mode observer. In Ref. [20] a sliding mode observer estimates both the rotor speed and the rotor position. In other proposals these variables are estimated by using adaptive reference models (see Refs. [21,22]). Sliding-observers can be considered as linear high gain observers where the correction term introduces a
constant high gain. These observers present chattering. For this reason, the estimated position is obtained from filtered EMF. The low pass filter used to filter the EMF deteriorates the system performance. Our proposal, introduces a correction term where a nonlinear gain is used to guarantee the convergence of the estimation error and improve the observer performance. In Ref. [23] the authors propose a method for wind speed estimation. The mechanical torque is approximated by using a neural network identifier. Our proposal uses a simple technique for the estimation of the mechanical torque. In Ref. [24] a sensorless control scheme for PMSG with diode bridge rectifier is presented. In this scheme, when parameters change, the system does not work in the maximum power point. For this reason, in Refs. [25], authors analyze the system behavior under this undesirable condition. In our scheme, it is assumed that in order to obtain the maximum power transfer, the control strategy is calculated to track the maximum power point. In Ref. [26] an algebraic filter is used. It must be noted that in this case, fast transients can make the system unstable. In our proposal, the dynamic model of the PMSG is considered in the design of the estimation algorithm. In Ref. [27] an unscented Kalman filter algorithm is proposed. This algorithm is based on Taylor's linearization technique. Our estimator design is based on a nonlinear technique. It is well-known that Taylor's linearization guarantees convergence in a small region around the equilibrium point. However, the PMSG model is highly nonlinear. For this reason it is better to use a nonlinear technique to build the estimator, such as it is proposed in this work.

In this work a mechanical sensorless strategy for controlling a WECS consisting of a wind turbine, a PMSG and two CSCs connected in a back-to-back configuration is proposed. The control strategy is designed for MPPT operation by controlling the PMSG torque. The controller uses estimates of the rotor speed, rotor position and the PMSG electrical torque. The estimates are obtained via a nonlinear observer [28]. This observer uses the measurement of the electrical variables. The proposed nonlinear observer consists of two terms. The first term copies the system dynamics. This is the prediction term. A correction term is added to the prediction term. The correction term includes a nonlinear gain matrix. This paper contains a criterion for designing the nonlinear gain matrix. Simulation results are presented in order to show the performance of the whole system.

The rest of the paper is organized as follows. In Section 2, the system is described. The PMSG model and the nonlinear observer used to estimate the mechanical variables are introduced in Section 3. Section 4 contains a brief description of the wind turbine model and the MPPT control method. Different simulation tests for validating the system performance are included in Section 5. Finally, conclusions are drawn in Section 6, and the criterion for designing the observer’s nonlinear gain matrix is presented in the Appendix section at the end of this paper.

2. System description

In Fig. 1 the WECS under study is shown. The turbine and PMSG axes are directly coupled (no gearbox is used), consequently both axes rotate to the same speed (ωm). The wind power captured by the turbine is transformed into electrical power by the PMSG. Then, this energy is transferred to the grid via a power conversion system. This power conversion system consists of two three-phase current converters connected in a back-to-back configuration with a coupling inductor (Lq) [29]. The generator side converter works as a CSR [1,29], controlling the power flux extracted from the turbine and transferred to the grid. The grid side converter works as a CSI [1,29], converting the energy stored in Lq, in a three phase current synchronized with the grid voltage. Both converters operate with Space Vector Modulation (SVM) [1,29,30]: m and θ represent the modulation index and the reference angle, respectively, used to control the CSR modulation. Whereas, m and θ represent the modulation index and the reference angle used to control the CSI modulation.

Since the current high frequency components flowing to the generator must be attenuated, the CSR and the PMSG are connected through an LC filter (LC in Fig. 1). The same concept is used for connecting the CSI and the grid (see LC). In order to implement the CSI controller, three phase grid voltage vG, three phase grid current iG and DC-link voltage Vdc (in the rectifier side) are measured, as shown in Fig. 1. It must be remarked that the CSI control is not the focus of this paper. For this reason, a simple control scheme for injecting current satisfying unity power factor condition is used. However, it is possible to design another strategy. For instance, a strategy proposing to inject reactive power to the grid could be employed [1,29,31]. The three phase measurements are transformed into a two-axes reference frame aβ. The grid voltage, in this new reference frame (vG), is transformed into polar coordinates and is represented by its magnitude |vG| and its angle θG. The CSI-SVM uses the modulation index m = Vdc/|vG| as reference [1].

It must be noted that the grid and inverter output currents are not in-phase, because the LC filter introduces a phase shift. In order to correct this deviation, CSI reference angle θ represents calculated grid voltage angle θ and a PI controller output, whose input is the cross product between current and voltage vectors (iα × vβ). This product is equal to zero when the current and voltage are in-phase.

A saturation block limits the PI output to −π/2 < θ < π/2.

The CSR controller uses the measurements of the generator currents and voltages (vG, iG, respectively) and its angle θG as reference frame. For this reason, the measurements are transformed into the aβ reference frame by using the Clarke’s transformation. In this way, variables vαG and iαG are obtained.

The goal of the “MPPT Control” block is make the turbine operate on the curve corresponding to the maximum power point transfer (MPPT) [1]. To this end, PMSG electrical torque Tm will be controlled to achieve that the turbine axle rotates to the optimal speed, irrespectively of the wind speed value. In order to obtain a good performance, the following variables should be fed back to the MPPT controller: the PMSG rotor speed (ωm), the PMSG rotor position (θm) and the PMSG electrical torque (Tm). Nevertheless, it is very important to remark that in our proposal, mechanical sensors will be avoided with minimal performance impact. To this end, estimated values of the rotor speed, the rotor position and the electrical torque (ωm, θm and Tm) will be obtained through a nonlinear observer [28], whose inputs are the electrical variables vG, iG and their derivatives (see block “Obs.” in Fig. 1).

3. PMSG model and nonlinear observer

In aβ reference frame, the non-salient pole PMSG model is described as [27]:

\[
\begin{align*}
\frac{d\omega_m}{dt} &= \alpha_{m2} - \alpha_{m1} \omega_m \\
\frac{d\theta_m}{dt} &= \frac{1}{J_t} \omega_m - \frac{1}{J_t} T_m - \frac{B}{J_t},
\end{align*}
\]

with
\[
Te = \frac{3}{2} P \lambda_M \left[ -i_G \sin(\theta_m) + i_G \cos(\theta_m) \right], \tag{2}
\]

where \(i_G\), \(v_G\), \(R\) and \(L\) are the generator currents and voltages in \(\alpha\beta\) reference frame; the stator resistance and inductance, respectively. \(\lambda_M\) is the rotor permanent magnet concatenated flux; \(\theta_m\) and \(\omega_m\) are the rotor position and speed; \(T_m\) is the turbine mechanical torque applied to the generator rotor, and \(T_e\) is the generator electrical torque; \(P\) is the number of pole pairs; \(J\) and \(B\) are the moment of inertia and viscosity coefficients of the combined turbine-generator set. An extended nonlinear observer [28] is used to estimate the mechanical variables (\(\omega_m\), \(\theta_m\)) and the generator electrical torque (\(T_e\)) (see block “Obs.” in Fig. 1). The observer consists of two terms. The prediction term copies the PMSG dynamic model and the correction term uses the measured currents \(i_{G\alpha\beta}\).

In order to avoid to use a torquemeter, turbine torque \(T_m\) is added to the model (Eqn. (1)) as a new state that will be estimated by the nonlinear observer. By assuming that the torque dynamics is slow, its dynamic model results:

\[
\frac{dT_m}{dt} = 0. \tag{3}
\]

The extended nonlinear observer is given by Ref. [28]:

\[
\begin{bmatrix}
\frac{d\theta_m}{dt} \\
\frac{d\omega_m}{dt} \\
\frac{d\hat{i}_G}{dt} \\
\frac{d\hat{i}_G}{dt} \\
\frac{d\hat{T}_m}{dt}
\end{bmatrix} = \begin{bmatrix}
\hat{T}_m - B\hat{\omega}_m - \frac{3P\lambda_M}{J} \left[ -i_G \sin(\theta_m) + i_G \cos(\theta_m) \right] \\
\frac{R}{L} \hat{i}_G - \frac{\lambda_M}{L} \hat{\omega}_m \sin(\hat{\theta}_m) - \frac{1}{L} v_G \\
\frac{R}{L} \hat{i}_G + \frac{\lambda_M}{L} \hat{p}_m \cos(\hat{\theta}_m) - \frac{1}{L} v_G \\
0
\end{bmatrix} + \gamma \begin{bmatrix}
\hat{i}_G - \hat{i}_G \\
\hat{i}_G - \hat{i}_G
\end{bmatrix}, \tag{4}
\]

with

\[
\gamma = \begin{bmatrix}
k_1 \sin(\hat{\theta}_m) - k_2 \cos(\hat{\theta}_m) \end{bmatrix}, \quad G = \begin{bmatrix}
G_{11} & G_{12} \\
G_{21} & G_{22} \\
G_{31} & G_{32} \\
G_{41} & G_{42}
\end{bmatrix}, \quad \Gamma = \begin{bmatrix}
-\frac{L}{\lambda_M} \cos(\hat{\theta}_m) & -\frac{L}{\lambda_M} \sin(\hat{\theta}_m) \\
\frac{P_{\omega_m}}{\lambda_M} & \frac{P_{\omega_m}}{\lambda_M} \\
\frac{-L}{\lambda_M} \sin(\hat{\theta}_m) & \frac{L}{\lambda_M} \cos(\hat{\theta}_m) \\
0 & 0 & 0 & 0
\end{bmatrix}.
\]
using calculated currents electrical torque as:

\[ T_e \approx \frac{1}{2} \rho \omega_m \tilde{\beta} \sin(\hat{\beta} m) + i_G \cos(\hat{\beta} m), \]

(5)

In the Appendix at the end of this paper the method used to obtain the observer's gain matrix \( \Gamma \) is presented.

### 4. Wind turbine model and MPPT control

The extracted wind power by a horizontal axle turbine can be calculated as [1]:

\[ P_m = 0.5 \pi \rho C_p(\lambda, \tilde{\beta}) \tilde{\beta}^2 v_w^3, \]

(6)

where \( r \) represents the wind turbine blade radius, \( v_w \) represents the wind speed, \( \rho \) represents the air density, \( \lambda \) is a coefficient including the tip speed of blade and wind speed rate. This coefficient is given by \( \lambda = \frac{\omega_m}{v_w} \), \( \tilde{\beta} \) represents the pitch angle and \( C_p(\lambda, \tilde{\beta}) \) is the turbine power coefficient. The turbine transfers into the generator a mechanical power \( P_m \). As a consequence, the mechanical torque applied by the turbine results \( T_m = P_m/\omega_m \). Fig. 2 illustrates (in per unity (pu)), the curves of mechanical torque as a function of \( \omega_m \), and parameterized as a function of the wind speed, for \( \tilde{\beta} = 0 \). For each \( v_w \) value, the maximum extracted power is a function of \( \omega_m \). Then, in order to extract the maximum power of the wind turbine, the mechanical torque must track the maximum power curve (dashed curve in Fig. 2). Then, the optimal torque value becomes:

\[ T_{mopt} = K_{opt} \omega_m^2. \]

(7)

where \( \tilde{\omega}_m, \hat{\theta}_m, \hat{i}_{Gc}, \) and \( \hat{i}_{Gq} \) stand for the estimation of the generator states defined in (1) and \( \hat{T}_m \) represents the estimated mechanical torque. By replacing estimated rotor position \( \hat{\theta}_m \) in Eqn. (2) and using calculated currents \( i_{Gc} \) and \( i_{Gq} \), it is possible to estimate the electrical torque as:

### Table 2

<table>
<thead>
<tr>
<th>Grid</th>
<th>CSR – CSI</th>
<th>LCR</th>
<th>LCI</th>
<th>PI_Lc</th>
<th>PI_Lq</th>
<th>PI_m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>VRL = 381Vrms; IR = 50 Hz</td>
<td>SVM = 10 kHz; Ldc = 10 mH</td>
<td>LR = 3 mH; CR = 0.5 mF; RR = 4Ω</td>
<td>Li = 1 mH; Ci = 5 mF; RI = 1 Ω</td>
<td>K_v = 1 x 10^{-4}; K_i = 5 x 10^{-2}</td>
<td>K_v = 1 x 10^{-4}; K_i = 5 x 10^{-2}</td>
</tr>
</tbody>
</table>

The Appendix at the end of this paper the method used to obtain the observer's gain matrix \( \Gamma \) is presented.
Fig. 4. Simulation of the system (see Fig. 1). Change of wind from 1 to 0.8 pu: (a) $T_m$ and $\bar{T}_m$; (b) $\omega_m$; (c) $T_e$ and $T_{e,ref}$.

Fig. 5. Simulation of the system (see Fig. 1). Change of wind from 1 to 0.8 pu: (a) $\omega_m$ and $\bar{\omega}_m$; (b) $\theta_m$ and $\bar{\theta}_m$; (c) $T_e \approx T_r$. 
The constant value $K_{opt}$ can be obtained using the turbine nominal data.

The block “MPPT Control” is shown in Fig. 3. This block controls the CSR in the WECS shown in Fig. 1. It must be noted that when $\left(\frac{d\omega_m}{dt}\right) = 0$ (i.e. steady state) from the PMSG model (Eqn. (1)), the following equation is obtained:

$$T_e|_{\omega_m=0} = T_m - B\omega_m.$$  \hspace{1cm} (8)

By combining Eqn (7) and (8), and using estimated angular speed $\hat{\omega}_m$ (4), the reference electrical torque for MPPT can be calculated as:

$$T_{ref} = K_{opt} \omega_m^2 - B\omega_m.$$ \hspace{1cm} (9)

It must be noted that to build this reference the speed estimated with the observer was employed, consequently the speed sensor is not needed. This reference must be tracked to guarantee operation on the maximum power curve. The reference is compared with the estimated electrical torque (Eqn. (5)) and the generated error is a PI input, whose output is CSR modulation index $m_G$, as shown in Fig. 3.

As for CSI (see Fig. 1), a scheme to guarantee that the machine delivers power satisfying the unity power factor condition is proposed. By using estimated rotor position $\hat{\theta}_m$, the internal EMF electrical angle estimate can be calculated as $\hat{\theta}_e = P\hat{\theta}_m$. This angle is considered the initial reference needed for generating the CSR reference angle $\theta_G$. To achieve unity power factor current injection ($i_G \pi$ - in-phase with $v_G$), $\theta_e$ is added to the output of a PI controller whose input is given by the cross product between current and voltage vectors $i_G \times v_G$ (see Fig. 3). Nevertheless, this condition can be modified to operate with another power factor value [1,29].

The block ‘Sat.’ limits the PI output to $-\pi/2 < \theta < \pi/2$.

5. Simulation results

In what follows, simulation results obtained for the system described in Fig. 1 are shown. These results were obtained with MATLAB®/SIMULINK®/SimPowerSystems® library [33]. The nominal power of the simulated system is $P_{nom} = 10$ kW. The PMSG [32], wind turbine and observer parameters are listed in Table 1. To reduce the simulation time, in order to visualize the dynamics details of the system variables, a low value of the moment of inertia was assumed. The parameters of the conversion system (CSR and CSI converters) and LCR and LCI filters are shown in Table 2. The SVM of both converters was implemented with a sample time $T_s = 1/f_{SVM}$ [30]. In addition, the parameters of the PI controllers of both converters are also shown in Table 2. In a first test, speed wind $\nu_w$ was changed from 1pu to 0.8pu (from 12 m/s to 9.6 m/s). As a result, wind turbine power $P_m$ decreases abruptly [see (6)]. This leads to a decrease in the mechanical torque, since $T_m = P_m/\omega_m$. Then, the Turbine-PMSG axle decelerates and its speed ($\omega_m$) decreases. It must be noted that the speed value is estimated by the observer. Consequently, the reference electrical torque (9) decreases as well (see “Control PMP” Figs. 1 and 3). Then, the controller causes a reduction of the electrical torque generated by the PMSG. Thus, extracted current ($i_G = [-i_G \sin(\theta_G) + i_G \cos(\theta_G)]$) is reduced as

![Fig. 6. Simulation of the system (see Fig. 1). Change of wind from 1 to 0.8 pu: (a) $v_{Gabc}$; (b) $i_{Gabc}$; (c) $v_{Rabc}$; (d) $i_{Rabc}$.](image)
well, and simultaneously the mechanical torque is increased. This transient behavior ceases when a new steady state point is reached. This new steady state point coincides with the intersection between the turbine torque-speed curve ($v_w = 0.8$ pu) and the maximum power curve. In Fig. 2 the arrowed trajectory shows the described behavior. Figs. 4–6 show the results obtained when a wind step was applied. Fig. 4(a) illustrates mechanical torque $T_m$ behavior and its estimate $\hat{T}_m$. There is an error because the mechanical torque was assumed slowly varying (see Eqn. (3)). However, if necessary, a more complex model can be included. For instance, a higher-order mechanical torque derivative term could be assumed equal to zero. In such a case, the mechanical torque will be described by a higher-order dynamical system.

Fig. 4(b) shows the change in mechanical speed $\omega_m$ from the nominal value $\omega_{m,\text{nom}} = 72.52$ rad/s to the steady state value $\omega_m = 58.01$ rad/s. This value corresponds to the turbine maximum power point when it operates with a wind speed equal to $v_w = 0.8$ pu (see Fig. 2). In Fig. 4(c), both electrical torque $T_e$ and its reference torque (Eqn. (9)) are shown. It can be seen that the CSR commutation includes a high frequency ripple in the electrical torque. In addition, it can be noted that the electrical torque abruptly decreases at the beginning because the generator active power also decreases. However, the behavior of $P_{\text{ImG}}$ allows to reach the new steady state value (Eqn. (8)) close to the mechanical torque value.

Actual and estimated values needed to implement the control law are shown, in detail, in Fig. 5. In Fig. 5(a) the mechanical speed $\omega_m$ and its estimate $\hat{\omega}_m$ are shown. In Fig. 5(b) rotor position $\theta_m$ and its estimate $\hat{\theta}_m$ are shown. In (c) $T_e$ and its estimate $\hat{T}_e$ are shown. It can be seen that the observer produces a good estimation of the mechanical angle. This angle is used to obtain $T_e$ (Eqn. (5)), then the electrical torque estimation error is small. The maximum error between the actual mechanical speed and the estimated speed is approximately 0.1 rad/s. Taking into account that the nominal speed is 72.52 rad/s, this error is very small. In addition, this error is corrected by the observer in a short time. Consequently, the control is not affected in a significative way. Thus, it can be concluded that the slowly varying assumption for the mechanical torque does not deteriorate the control performance.

In Fig. 6(a) grid voltage $v_{\text{grid}}$ is shown; and in Fig. 6(b) the behavior of three-phase current ($i_{\text{grid}}$) is shown. In this figure the thick curves correspond to phase “a” both for the PMSG and the grid. It can be seen that, after the wind speed step, has been applied both PMSG current and voltage value are decreased. The current

![Fig. 7](image-url)
supplied to the grid is decreased as well. It is obviously a consequence of the turbine power reduction. It can be noted that both the PMSG and the grid operate with unity power factor, i.e. currents and voltages are in-phase. It is clear that the system performance is good and the control strategy used to generate the reference angles for CSI and CSR ($\theta_R$ and $\theta_G$, respectively) works as expected.

In a second test, a realistic wind speed profile is applied to the system. Fig. 7(a) shows the wind speed profile. This profile is obtained using the von Karman turbulence model [34]. For the time interval in which this wind profile is applied, Fig. 7(b) shows the actual and the estimated mechanical speed, Fig. 7(c) shows the actual and the estimated mechanical torque, and 7(d) shows the electrical torque and its reference. Fig. 8 shows the estimation errors. It can be seen that the proposed controller performs very well. Fig. 8(b) shows the mechanical torque error. This error is less than 5% of the nominal torque. Meaning that the error provoked by assuming a slowly varying torque is broadly tolerable for this application, and consequently a more complex model is not needed. In addition, it can be observed that the other estimation errors are very small.

6. Conclusions

In this work a mechanical sensorless control strategy for a speed variable WECS based on PMSG was introduced. This strategy uses an extended nonlinear observer to estimate the mechanical variables needed to control the PMSG electrical torque. This torque is controlled to track the wind turbine maximum power point.

The main goal behind the proposed strategy is to avoid the use of mechanical sensors in the WECS. This is an important contribution to reduce cost and to increase reliability of the system, mainly in WECS including small size power turbine.

Simulations were introduced to show the performance of the proposed system. It was demonstrated that the system performance is similar to that obtained with controllers that require a mechanical sensor.

Acknowledgements

This work was partially supported by UNS, CONICET and ANPCyT, Argentina.
The PMSG model is given by:

\[
\dot{\omega}_m = \frac{\omega_m - \frac{\lambda_m}{L} P_{om} \sin(\theta_m) - 1}{L_o},
\]

\[
\dot{i_a} = \frac{R_o}{L_o} i_a - \frac{\lambda_m}{L_o} \omega \sin \theta - \frac{1}{L_o} V_a,
\]

\[
\dot{i_b} = \frac{R_o}{L_o} i_b + \frac{\lambda_m}{L_o} \omega \cos \theta - \frac{1}{L_o} V_b,
\]

\[
\dot{T_m} = 0, \quad \dot{y} = [i_a, i_b]^T.
\]

In the new coordinates, the PMSG model is represented by

\[
\rho(\xi) = |m_1, m_2, 0, 0, 0|^T = \begin{bmatrix} m_1 & m_2 & 0 & 0 & 0 \end{bmatrix} \in \mathbb{R}^4
\]

(13)

\[
\begin{bmatrix}
\dot{m}_1 \\
\dot{m}_2 \\
dot{m}_3 \\
\dot{m}_4 \\
\dot{m}_5
\end{bmatrix} = \begin{bmatrix}
-k_k \xi_3 \xi_4^2 - k_k \xi_4 \xi_5^2 - \sqrt{\xi_1^2 + \xi_2^2} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} - \frac{k}{J} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} \\
-k_k \xi_4 \xi_5^2 - k_k \xi_4 \xi_5^2 - \sqrt{\xi_1^2 + \xi_2^2} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} - \frac{k}{J} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} \\
-k_k \xi_3 \xi_4^2 - k_k \xi_4 \xi_5^2 - \sqrt{\xi_1^2 + \xi_2^2} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} - \frac{k}{J} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} \\
-k_k \xi_4 \xi_5^2 - k_k \xi_4 \xi_5^2 - \sqrt{\xi_1^2 + \xi_2^2} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} - \frac{k}{J} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} \\
-k_k \xi_3 \xi_4^2 - k_k \xi_4 \xi_5^2 - \sqrt{\xi_1^2 + \xi_2^2} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}} - \frac{k}{J} \frac{\xi_1}{\sqrt{\xi_1^2 + \xi_2^2}}
\end{bmatrix}
\]

(14)

and \(k_k = 3\phi/2J; \ k = \lambda_m/L; \ v = [v_a, v_b].\)

The following observer is proposed for estimating the states in the transformed coordinates.

\[
\dot{\hat{\xi}} = A_e \hat{\xi} + \rho(\hat{\xi}) + V_v + G_e (y - \hat{y}) + l_a.
\]

\[
y = C_e \hat{\xi},
\]

where \(G_e = [G_0]^T; \ C_e \in \mathbb{R}^{4 \times 2}; \ I_a = [I_0, I_0]^T; \ I_a \in \mathbb{R}\) is the adaptation law.

\[
\dot{e}_c = [A - GC_0]^T e_c + \Delta \rho - l_a.
\]

(16)

The adaptation law and the gain matrix must be chosen to guarantee the convergence to zero of the estimation error. A Lyapunov candidate function is chosen. This function is given by:

\[
V = e_c^T P e_c, \quad P = \begin{bmatrix} P & 0 \\
0 & 0 \end{bmatrix}.
\]

(18)

The Laypunov function time derivative results

\[
V \leq -e_c^T (A_e^T P + P A_e) e_c + 2e_c^T P \Delta m - 2l_a e_c^T.
\]

By choosing matrix \(Q\) to satisfy the following Lyapunov equation

\[
A_e^T P + P A_e = -Q, \quad Q > 0.
\]

(20)

then, the following equality is obtained.

\[
V = -e_c^T Q e_c + 2e_c^T P \Delta m - 2l_a e_c^T.
\]

(21)

This equation suggests to use the following adaption law:

\[
l_a = h^T (\hat{\xi} P e_c); \ \text{con} \ h^T = [m_{11}, m_{22}, 0, 0]. \]

Then, the following inequality is satisfied for the Lyapunov function time derivative.

\[
V \leq (-q + 2p L_1 + 2p L_2 \|\xi_3\|) e_c^T.
\]

(22)
In this inequality, $q$ is the minimum eigenvalue of $Q$. $p$ is the maximum eigenvalue of $P$. $L_1$ and $L_2$ are the Lipschitz constant of $m_1$ and $m_2$, respectively.

By assuming $||\xi^0||$ is bounded, when $\sigma = -q + 2PL_1 + 2PL_2||\xi^0|| < 0$ the estimation error converges to zero. It must be noted that $\epsilon_1$ and $\epsilon_2$ cannot be used in the adaptation law, because $\hat{\xi}_1$ and $\hat{\xi}_2$ are not measurable. In original coordinates, the adaptation law built from the measured variables results:

$$I_a = -k_1\sin\left(P\theta_m\right)\left(i_a - \tilde{i}_a\right) + k_2\cos\left(P\theta_m\right)\left(i_b - \tilde{i}_b\right). \quad (23)$$

Then, coming back to the original coordinates the observer description is given by (4).

References