Brief Papers

Servomechanism Controller Design of Web Handling Systems

Weixuan Liu and E. J. Davison

***Abstract*—In traditional web handling processes, web tension and speed are typically controlled assuming that the web system consists of a number of single-input–single-output systems. This assumption often results in large interactions occurring in the closed-loop system between the control loops and, hence, results in high-quality control being difficult to achieve. In this paper, the control of the web handling processes is treated as a multivariable servomechanism problem. Two types of controller design-the "perfect control servomechanism controller" and the "tuning regulator" are studied and implemented on an industrial web machine. The experimental results obtained show that these controllers provide excellent tension and speed response compared with conventional controllers used in web systems. In particular, it is shown that the "tuning regulator" approach is only marginally worse than the "perfect control servomechanism controller" approach, despite the fact that it does not require a mathematical model of the web process, and is simpler to implement.**

***Index Terms*—Perfect control, robust servomechanism control, servo-compensator, tuning regulator control, web handling systems.**

I. INTRODUCTION

In multivariable industrial controller design, three approaches can be taken.

1) Assume the system consists of a set of single-input–single-output (SISO) control loops and design each control loop independently of the others using SISO methods.
2) Define a mathematical model of the system using either analytical or identification methods, and then apply any of the well-known multivariable controller synthesis methods to design a multivariable controller for the system.
3) Apply some type of multivariable tuning controller design, e.g., the approach of [1], in which no mathematical model of the system is required and which only requires carrying out steady-state experiments on the system.

In the case of 1), this approach has the advantage of simplicity and is often used, but the disadvantage is that the resulting controller may be poor due to ignored interaction effects. In the case of 2), the main disadvantage of the method is in the effort required in the construction of a suitable mathematical model of the system, or in the difficulty in carrying out identification experiments, and in the fact that there is no guarantee that the resultant model obtained is "sufficiently accurate" for controller design.

The main purpose of this paper is to compare the effectiveness of the above methods of controller design on an actual industrial system. The system chosen is a Rotoflex industrial web handling system (approximate size $7' \times 6' \times 4'$), with roller width around 0.3 meters and operating speed under 5 m/s. In particular, it was desired to improve on the controller performance over an existing controller design obtained using approach 1).

A comprehensive study of web tension control problem can be found in [2]. Recently, new approaches to web tension control have been studied such as nonlinear control used in [3] and fault-tolerant control in [4]. In this paper, only linear time-invariant (LTI) controllers will be studied.

A. Background Knowledge

In industry, paper, plastic, and other elastic thin materials are often used in the manufacturing of commercial products by using a continuous process. In this case, the paper or other material is typically unrolled from a large roll using a series of rollers and a rewinder, forming what is called a web.

To produce an end product from a raw web material, such as from a paper machine or a film extruder, two kinds of processes are involved: Web converting and web handling. Web converting includes all those processes which are required to modify the physical properties of the web material such as coating, slitting, metalizing, drying, and embossing, etc. The web handling processes, on the other hand, consists of those processes which are associated with the transportation aspects of the web. The main purpose of the web handling process is to transport web with maximum throughput (speed) and with minimum damage [5]. To achieve this, web tension control is crucial because of the following reasons.

1) Web tension affects the geometry of the web, such as the apparent length and width of the web.
2) High web tension prevents the loss of traction on the rollers; however too high web tension will cause a web break to occur.
3) Web tension control helps to reduce wrinkling. In particular, high process tension will help decrease the wrinkling caused by a misalignment of rollers; however, excessively high tension will cause more wrinkling to occur on very thin materials. Hence, appropriate web tension control is very important.

Manuscript received June 19, 2001. Manuscript received in final form October 25, 2002. Recommended by Associate Editor F. Doyle.

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Digital Object Identifier 10.1109/TCST.2003.810377

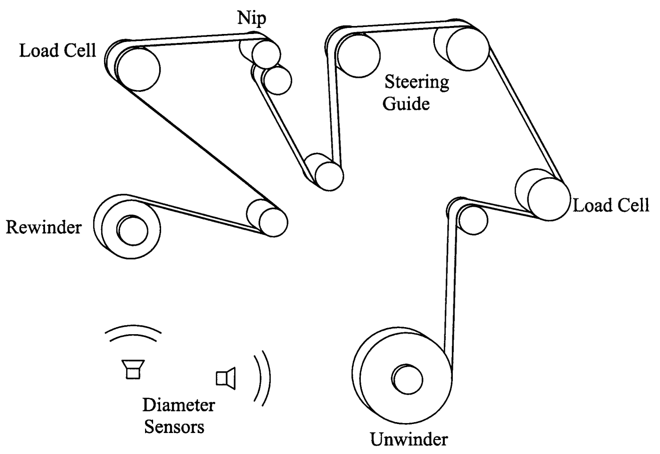


Fig. 1. Overview of the Rotoflex machine.

4) Web tension affects the wound-in tension and the shape of the final product roll and, hence, the roll quality.

For these reasons, it is essential in web handling to control the web tension at a desired value as closely as possible. Normally, web tension should be set at 10%–25% of the web's yield strength and should be kept within 10% of this value during the system's steady running state and 25% of this value at speed setpoint changes [5].

Almost all the components of a web machine influence the web tension. We will now examine the properties of the web machine which was used in this study.

B. Rotoflex Web Handling Machine

As a typical small size web handling machine, the Rotoflex web machine has all of the typical necessary components associated with a web handling process (see Fig. 1).

Rollers are essential parts of a web handling machine. In any machine, there are two type of rollers: 1) externally torque driven rollers such as the unwinder, rewinder and the nip roller and (2) the web driven rollers (idlers). These devices are also called "transport rollers" in industry because they are not intended to change the physical properties of the web. The traditional role of a "nipped roller" is to step the tension up or down between sections of processes and, hence, create different tension zones for different processes. In designing a controller for a web system, the nipped roller torque input and the wound roller torque inputs (rewinder and unwinder) provide multiple inputs for multivariable tension/speed control. In the web machine studied, these torque inputs are regulated by pulse-width modulation (PWM) drives. The torque outputs from the PWM drives can be either positive or negative and, hence, can either act as "drives" or "brakes" in web control. Besides the nip and winders, there are also some web driven rollers (idlers), which provide additional inertia to the web system.

The tensions of the web system are measured by load cells. To provide real-time monitoring of time varying information such as inertia of the unwinder and rewinder, there are two diameter sensors which measure the changing diameters of the unwinder and rewinder, in the Rotoflex web machine.

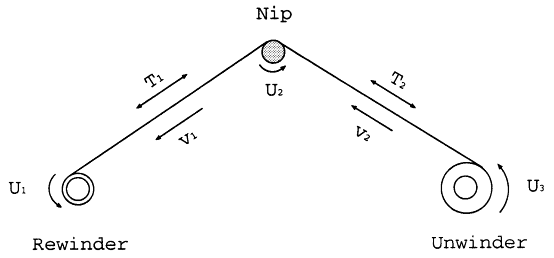


Fig. 2. Illustrative diagram of the Rotoflex machine.

In this paper, two controller design approaches: perfect robust servomechanism controller (PRSC) design [6] and tuning regulator controller (TRC) design [1] will be implemented on the Rotoflex web machine. In the case of the PRSC design, an analytical model of the web system is required, and a controller synthesis procedure, which is parameterized by a cheap control gain parameter $\epsilon > 0$, is used, which solves the robust servomechanism problem for a given class of tracking/disturbance signals (provided that a solution exists); this procedure has the additional property that "perfect control" occurs as $\epsilon \rightarrow 0$ in the resultant closed-loop system (ignoring nonlinear control signal saturation effects), provided that certain necessary conditions are satisfied (e.g., the plant is minimum phase). In the case of the TRC design, a controller synthesis approach is given, which again is parameterized by tuning parameters $\epsilon_P > 0$ and $\epsilon_I > 0$, and which has the advantage that a mathematical model of the system is not required to construct the controller, unlike the PRSC approach.

II. MODELING OF THE WEB MACHINE

Web modeling has been extensively discussed in previous works such as in [2] and [7]. In this paper, we will develop a web model only in the longitudinal direction.

The Rotoflex web machine is a two span machine as illustrated in Fig. 2, where U_1, U_2, U_3 denote control input torques, T_1, T_2 denote output tensions, and v_1, v_2 denote output web velocities, where v_1 is approximately equal to v_2. Let v denote the average velocity given by $1/2(v_1 + v_2)$.

The major components of this system consist of an unwinder, a winder, a nip and the web connecting them. In between the rewinder and the nip, and between the nip and the unwinder, there are a number of rollers as shown in Fig. 1. In constructing an analytical model for the system, these idlers are ignored, which introduces uncertainty in the resultant model obtained.

A. Reduced Order Analytical Model

The web material used in the experiments carried out in this study is paper, which has a very high elasticity modulus and, hence, can be considered as a stiff material. In this case, an analytic model of the web system is developed in [8] for this type of material.

On using the parameters of the Rotoflex web machine described in Table I, which are obtained from identification experiments on the web machine, we obtain the following reduced order model on ignoring the idlers and fixing the radius of the

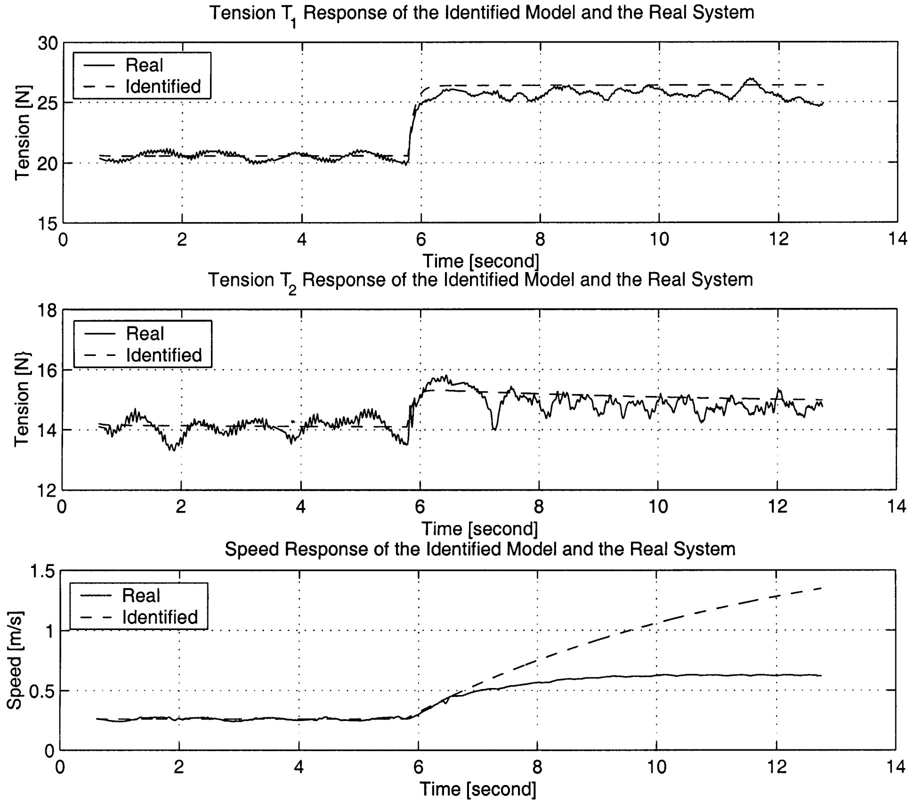


Fig. 3. Response of the open-loop experimental system versus the LTI reduced order low-frequency model (3) for the case of a step change in rewinder torque input U_1.

TABLE I
LIST OF IDENTIFIED PARAMETERS

parameter	definition	value
r_{0a}	Radius of the bare unwinder	0.0415m
r_b	Radius of transport roller (Nip)	0.0415m
r_{0b}	Radius of bare rewinder	0.0415m
J_{0a}	Inertia of bare unwinder	$0.0175kgm^2$
J_b	Inertia of transport roller	$0.0322kgm^2$
J_{0c}	Inertia of bare rewinder	$0.0234kgm^2$
b_a	Damping friction coefficient of unwinder	$1/254Nms$
b_b	Damping friction coefficient of transport roller	$1/165Nms$
b_c	Damping friction coefficient of rewinder	$1/258Nms$
e	Web thickness	2.39×10^{-5}m

unwinder r_a at 0.0833 m

$$\dot{v} = -0.168v + [0.480 \quad 0.846 \quad 0.412] \times [U_1 \quad U_2 \quad U_3]^T$$

$$\begin{bmatrix} T_1 \\ v \\ T_2 \end{bmatrix} = \begin{bmatrix} 0.0337 \\ 1 \\ -0.340 \end{bmatrix} v + \begin{bmatrix} 11.5 & -3.86 & -1.88 \\ 0 & 0 & 0 \\ 2.49 & 4.39 & -9.58 \end{bmatrix} \times \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}. \tag{1}$$

Since the actual Rotoflex system has filters of the structure

$$F(s) = \frac{1}{0.1s + 1} \tag{2}$$

connected to each of the tension outputs, such an addition of filters was also applied to (1) to obtain the following $n = 3$ model of the system (called model #1):

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -10 & 0 & 0.135 \\ 0 & -10 & -1.36 \\ 0 & 0 & -0.168 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 45.9 & -15.4 & -7.51 \\ 9.96 & 17.6 & -38.3 \\ 0.480 & 0.846 & 0.412 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

$$\begin{bmatrix} T1 \\ v \\ T_2 \end{bmatrix} = \begin{bmatrix} 2.5 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2.5 & 0 \end{bmatrix} \begin{bmatrix} x1 \\ x_2 \\ x_3 \end{bmatrix}. \tag{3}$$

The response of (3) is compared to the response of the actual plant by carrying out a set of open-loop rewinder torque step input experiments at the operating point $r_a = 0.0833$ m, and the plot given in Fig. 3 shows the difference between the responses of (3) and the response of the actual system for the case of a step input in the rewinder torque.

We observe from Fig. 3 that the ignored idlers have little effect on the the tension responses. A step input on rewinder torque causes almost the same amplitudes of tension increase and tension responses to occur for both (3) and the actual system. However, the speed responses of the identified model and the real system differ significantly.

A step input in rewinder torque causes approximately five times greater change in speed response for (3) than for the actual system. This is because the actual system has more friction than the model (3).

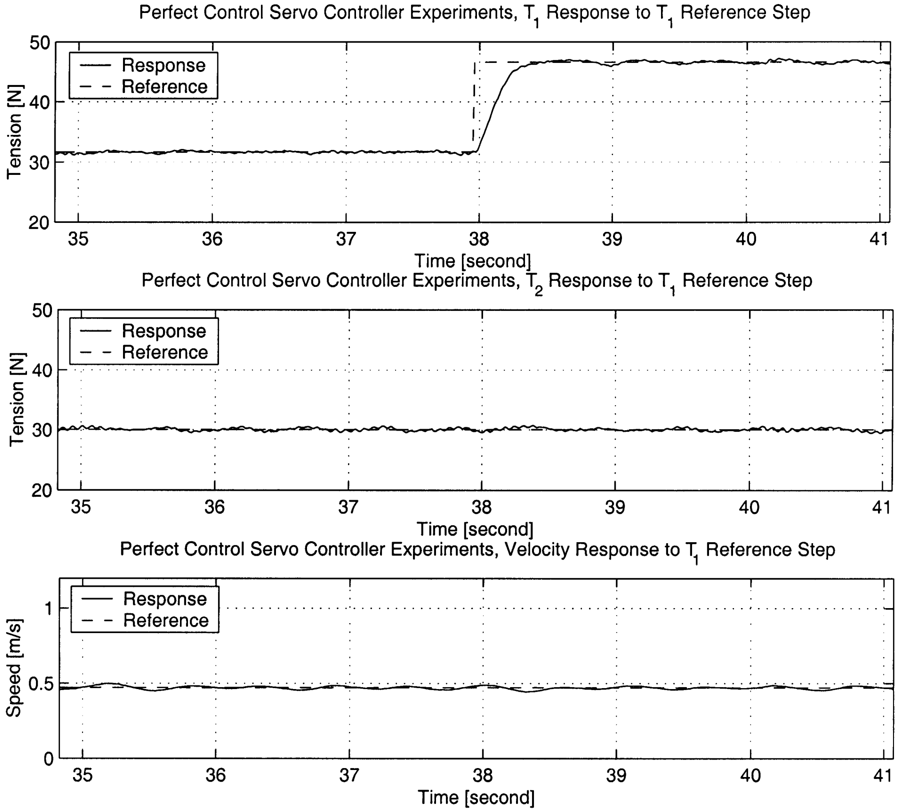


Fig. 4. Experimental output responses to tension T_1 step reference input (controller #1).

Overall speaking, (3) has a good match in tension responses with the actual plant, other than some cycling effects which occur in the actual system. It is conjectured that this cycling is due to the nonlinear effects which were ignored in (3).

For speed response, the main difference of the actual system and (3) is that the actual system has more damping, and a smaller gain than (3).

Two design methodologies: perfect control design [6] and tuning regulator design [1], will now be carried out.

III. PERFECT CONTROLLER DESIGN

A. Design of the Servomechanism Controller [6]

The reference signals to the system are assumed to be constant corresponding to the tension and velocity set points which are to be tracked. There are several main sources of disturbance to the web system.

1) Constant type offset signals: These signals include external disturbance sources such as which arise from various loading effects.
2) Periodic type disturbance signals: These signals include periodical disturbances associated with the mechanical vibration in the web system such as which arise from unbalanced rollers.
3) Stochastic type disturbance signals: These signals include signals such as the wound-in tension $T_w(t)$ from the unwinder.In the controller design to be carried out, it will be assumed that the disturbances are unmeasurable and constant.

Thus, the servocompensator can be simply chosen to be

$$\dot{\xi} = y - y_{\rm ref} \tag{4}$$

where $y = (T_1 \ v \ T_2)'$ is the plant output, $y_{\rm ref} = \left(T_1^{\rm ref} \ v^{\rm ref} \ T_2^{\rm ref}\right)'$ is the reference signal, and ξ is the servocompensator state.

It will now be assumed that the web system can be described by the LTI model given by (3). In this case, a stabilizing controller will be found to stabilize and give satisfactory transient response to the augmented system

$$\begin{bmatrix} \dot{x} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} x \\ \xi \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix} u$$
$$z = [\mathbf{0} \quad \mathbf{I}] \begin{bmatrix} x \\ \xi \end{bmatrix} \tag{5}$$

where the plant model $(\mathbf{C}, \mathbf{A}, \mathbf{B})$ is given by optimizing the "cheap control" performance index

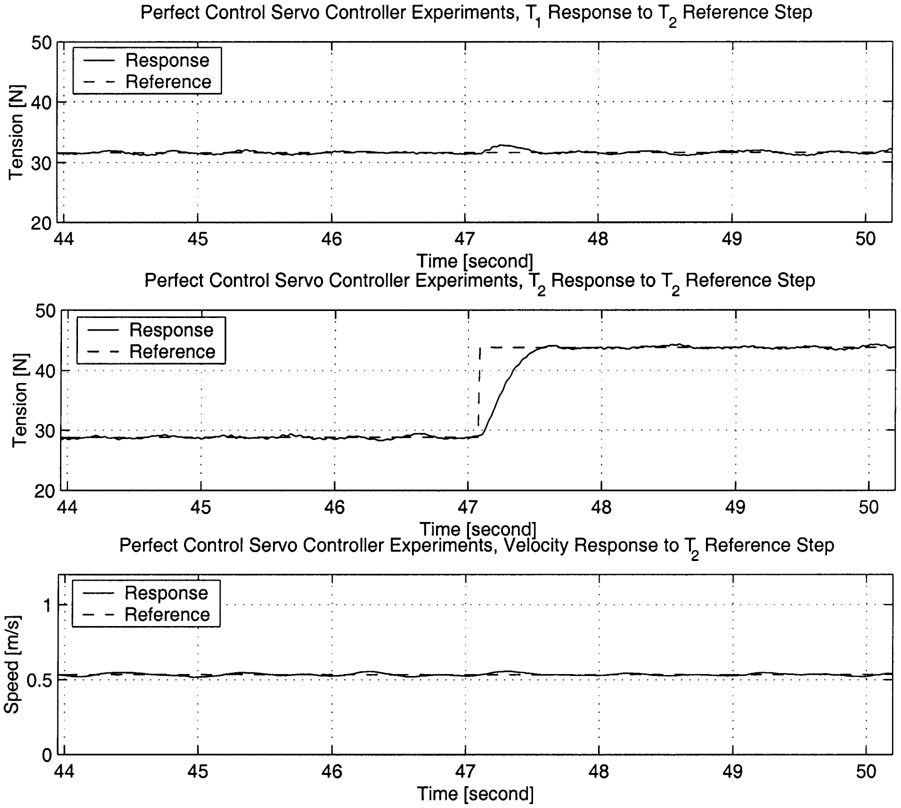
$$J = \int_0^\infty \{z'z + \epsilon u'u\} d\tau \tag{6}$$

where $\epsilon > 0$, to give a controller of the type [6]

$$\begin{aligned} \dot{\xi} &= y - y_{\rm ref} \\ u &= K_0 x + K_1 \xi. \end{aligned} \tag{7}$$

B. Experimental Results—Controller #1

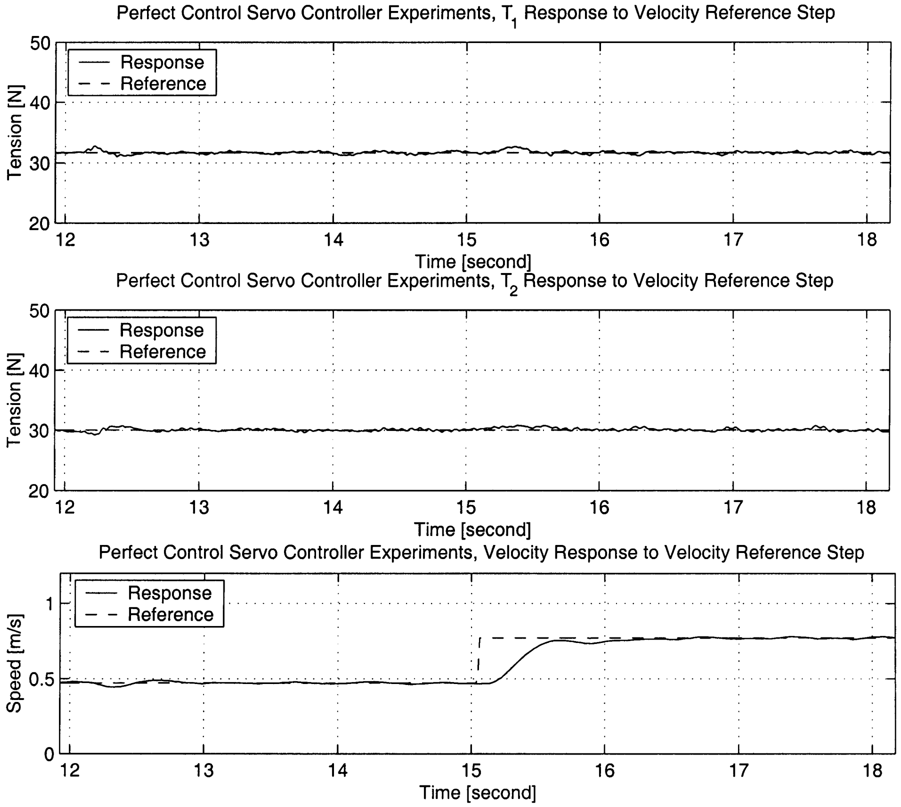
A controller was designed based on the above procedure, and the resulting controller is called controller #1. The following plots given in Figs. 4–6 give the experimental closed-loop re-


Perfect Control Servo Controller Experiments, T_1 Response to T_2 Reference Step

Perfect Control Servo Controller Experiments, T_2 Response to T_2 Reference Step

Perfect Control Servo Controller Experiments, Velocity Response to T_2 Reference Step

Fig. 5. Experimental output responses to tension T_2 step reference input (controller #1).


Perfect Control Servo Controller Experiments, T_1 Response to Velocity Reference Step

Perfect Control Servo Controller Experiments, T_2 Response to Velocity Reference Step

Perfect Control Servo Controller Experiments, Velocity Response to Velocity Reference Step

Fig. 6. Experimental output responses to velocity step reference input (controller #1).

sponse for the above controller obtained when the cheap control gain ϵ is chosen to be $1e-4$ for the case of step reference inputs to T_1^{ref}, v^{ref}, and T_2^{ref}, respectively. A complete description of this controller can be found in [8].

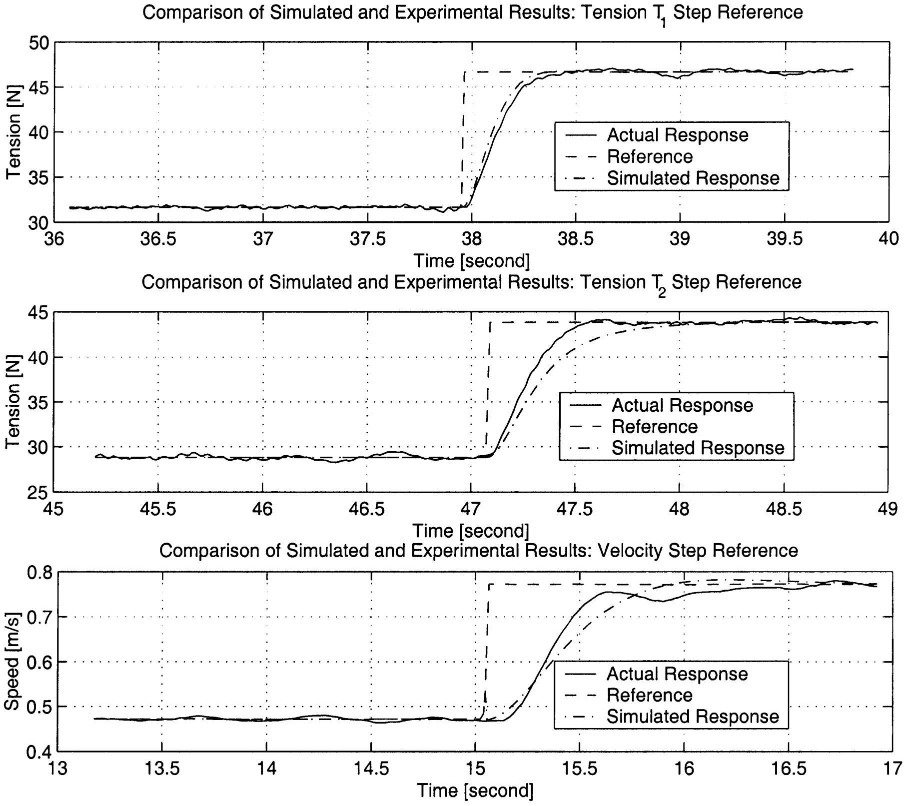


Fig. 7. Comparison of the closed-loop responses of the experimental results obtained using controller #1 and the theoretical results obtained using model #1 (with controller #1).

C. *Discussion of the Experimental Results Obtained From Controller #1*

1) *Transient Response and Interactions:* A very important criteria for controller design is the transient response time. As we can see from Figs. 4–6, the tension response has a settling time of approximately 0.3 s and a speed response of less than 1 s, which is much faster than the SISO proportional, integral, and derivative (PID) controllers commercially used in web system control. Another criteria for controller performance is the interaction between outputs, i.e., it is desired that tension set point change in one span should not cause variation in the response of the tension of the other span or the machine speed. Also, the speed set point change should cause as little variation in the tension responses as possible. As can be seen in Figs. 4–6, the largest interaction occurs when the speed set point is changed. However, in this case, the variation of tension is approximately $1N$ (corresponding to less than 5% of the tension set points), which is negligible by the industrial criteria of 25% of the tension set point values.
2) *Steady-State Variations:* The maximum variation of tension at steady state is approximately 1–$2N$ (corresponding to 2%–5% of the tension reference values), which is much lower than the required industrial criteria of 10% of the set point value.
3) *Comparison With Theoretical Results:* It is interesting to compare the experimental results of the actual machine with the results obtained from the simulation of the analytic model #1 to determine the effect of unmodeled uncertainties on the overall closed-loop system.

TABLE II
STEADY-STATE VALUE DIFFERENCE

Experiment	Steady State Value Difference		
	δT_1^{ss}	δv^{ss}	δT_2^{ss}
Torque Step Input δU_1=[0.6, 0, 0]^T	7.76	0.367	0.796
Torque Step Input δU_2=[0, 0.6, 0]^T	-1.58	0.378	0.780
Torque Step Input δU_3=[0, 0, -0.3]^T	1.32	-0.145	5.24

It is observed from Fig. 7 that the experimental results and the results obtained from simulation of the model #1 are very similar.

IV. TUNING REGULATOR DESIGN

From the previous controller implementation discussion, we recognize that a "good model" is very important in controller design. However, the process of finding such a model is a very tedious procedure, and a reliable model is always very difficult to obtain. Hence, a different methodology which bypasses the plant model construction aspect of controller design is appealing. If the plant is assumed to be LTI and open-loop stable, then it is shown in [1] that a tuning regulator design approach can be used, which requires no mathematical

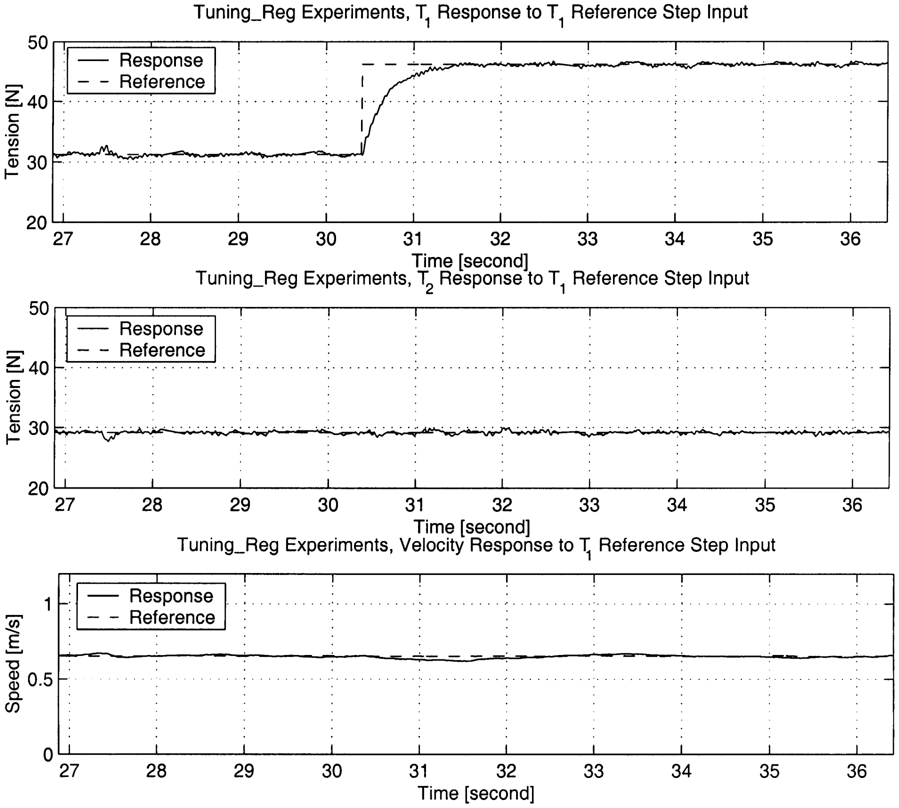


Fig. 8. Tuning control experiment: experimental responses to T_1 step reference input.

model of the system; it only requires that some steady-state experiments be carried out.

A. Preliminary Experiment

On using the approach of [1], it is assumed that the plant to be controlled can be described by an LTI model. In the case when this assumption cannot be satisfied, then the approach of [1] can be extended to nonlinear plants by applying gain-scheduling techniques, e.g., see [10].

In the case of the web system, the analysis carried out in [8] indicates that the web system is in fact a nonlinear system. However this analysis also indicates that the web system can be approximately described by a LTI model, and so this assumption will also be made here (as was done using the previous controller design approach). The analysis also indicates that the web system can be described by a model which is open-loop stable, as has been demonstrated by experiment, and so the assumptions required by the tuning regulator approach hold.

A set of three open-loop step torque input experiments with inputs δU_1, δU_2, and δU_3 was then carried out on the Rotoflex web machine for the case when $r_a = 0.0732\ \mathrm{m}$ and $r_c = 0.0853\ \mathrm{m}$; the corresponding steady-state values obtained under these step inputs are listed in Table II.

In web control, the main control objective is to track constant tension and constant velocity set points, and reject any constant unmeasurable disturbance in the system.

According to the results of [1], when the control objective is tracking/rejecting constant signals, the information obtained in Table II is only required in order to solve the servomechanism problem for the web system using the tuning regulator controller approach.

Definition: Given an asymptotically stable LTI system

$$\begin{aligned}\dot{x} &= Ax + Bu + E\omega \\ y &= Cx + Du + F\omega\end{aligned} \tag{8}$$

the steady-steady gain matrix of the system (8) for constant reference input signals is given by

$$G = -CA^{-1}B + D. \tag{9}$$

It then follows from [1] that there exists a servomechanism controller for the web system for constant reference/disturbance signals if and only if $\mathrm{rank}(G) = \dim(y)$, i.e., the number of inputs of the web system is at least equal to the number of outputs and the gain matrix G has a rank not smaller than the number of outputs.

For the web system used in this experiment, the gain matrix G can be determined from Table II [1]

$$\begin{aligned} G &= -CA^{-1}B + D \\ &= \begin{bmatrix} 7.76 & -1.58 & 1.32 \\ 0.367 & 0.378 & -0.145 \\ 0.796 & 0.780 & 05.24 \end{bmatrix} \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & -0.3 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} 12.9 & -2.64 & -4.39 \\ 0.611 & 0.630 & 0.484 \\ 1.33 & 1.30 & -17.5 \end{bmatrix} \end{aligned} \tag{10}$$

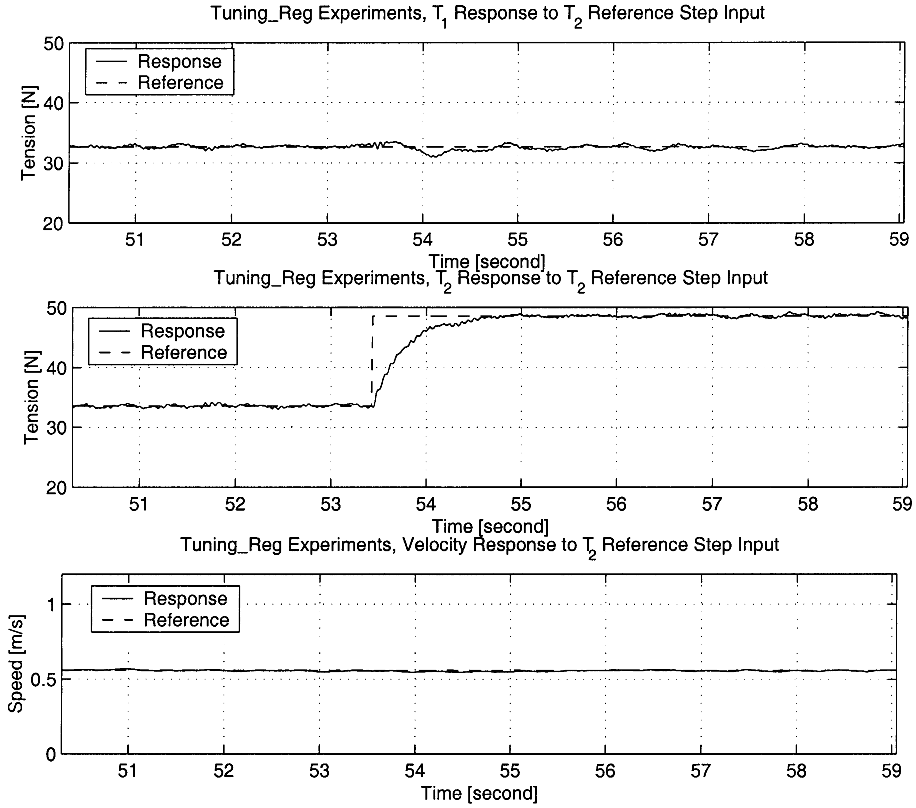


Fig. 9. Tuning control experiment: experimental responses to T_2 reference step input.

and, thus, there exists a solution to the servomechanism problem since

$$\operatorname{rank}(G) = 3 = \text{number of outputs}. \tag{11}$$

The tuning regulator is then given by [1]

$$u = \epsilon_P G^{-1} \begin{bmatrix} T_1^{ref} - T_1 \\ v^{ref} - v \\ T_2^{ref} - T_2 \end{bmatrix} + \epsilon_I G^{-1} \begin{bmatrix} \int_{t_0}^{t} (T_1^{ref} - T_1) dt \\ \int_{t_0}^{t} (v^{ref} - v) dt \\ \int_{t_0}^{t} (T_2^{ref} - T_2) dt \end{bmatrix} \tag{12}$$

where T_1^{ref}, v^{ref}, and T_2^{ref} are reference signals for tension T_1, velocity v, and tension T_2, respectively, and $\epsilon_P > 0$ and $\epsilon_I > 0$ are scalers found by using "on-line" tuning to achieve the best overall control effect.

B. Results Obtained From Experiments

On applying the controller (12) to the web machine, the following optimal values of parameters $\epsilon_P > 0$, $\epsilon_I > 0$ were obtained by carrying out one-dimensional online tuning, starting with parameter ϵ_P, then fixing the optimal value of ϵ_P obtained, and then varying parameter ϵ_I

$$\begin{aligned} \epsilon_P &= 0.4 \\ \epsilon_I &= 4. \end{aligned} \tag{13}$$

Some experimental responses of the resultant closed-loop system obtained using this controller are given in Figs. 8–10.

Behavior of the Tuning Regulator: From Figs. 8–10, it can be observed that the tension responses display satisfactory behavior with respect to both transient state and steady state, with a time constant less than 1 s. However, we also observe that the speed input response is somewhat oscillatory. This could be improved by adding a rate feedback term in the controller (12).

V. CONVENTIONAL CONTROL

It is of interest to compare the responses of the closed-loop system obtained using the controllers of Sections III and IV with the "conventional controller" (a PID controller tuned using SISO methods) normally used on the Rotoflex web system. Fig. 11 gives a representative response of the system controlled using a PID controller for a ramp change in the velocity. It can be seen that the response of the resultant system has significant interaction compared to the proposed controllers.

VI. CONCLUSION

In this paper, the control of an industrial web process has been carried out using two different controllers.

1) A "perfect robust servomechanism controller" based on an analytical model of the web system in conjunction with the measurement of certain parameters of the web system (Table I).
2) A "tuning regulator controller" based on some steady-state experimental measurements of the web system.

Both controllers produced excellent tracking control and disturbance regulation, and were superior to the conventional three-term controller presently used on the machine (which produced significant interaction effects). What is especially interesting, however, is the observation that the "tuning regulator

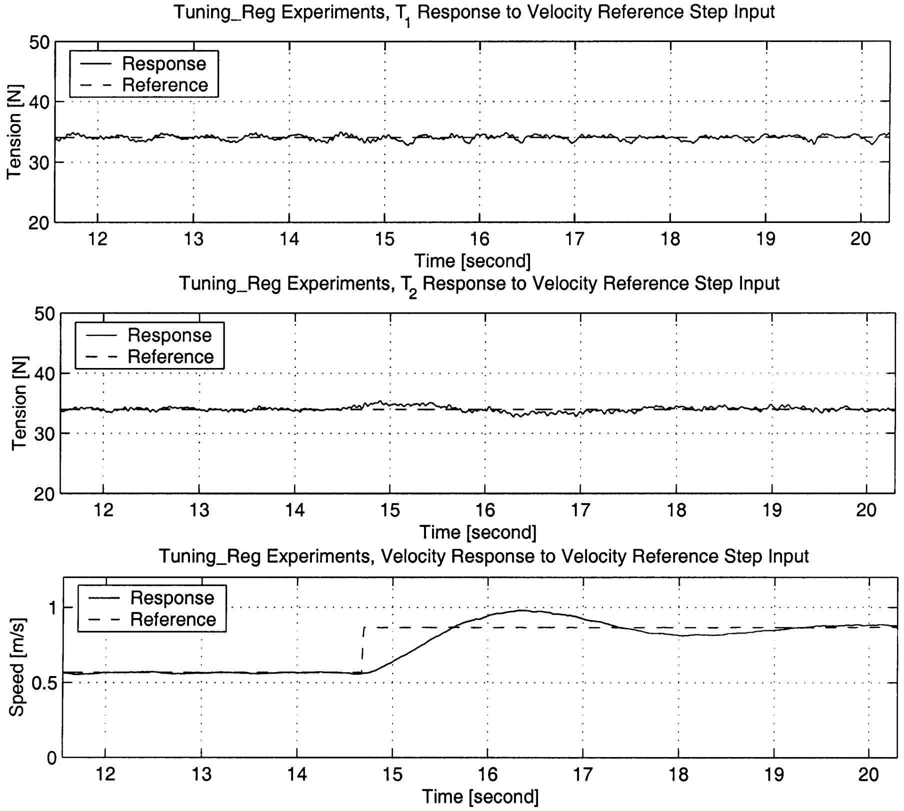


Fig. 10. Tuning control experiment: experimental responses to velocity reference step input.

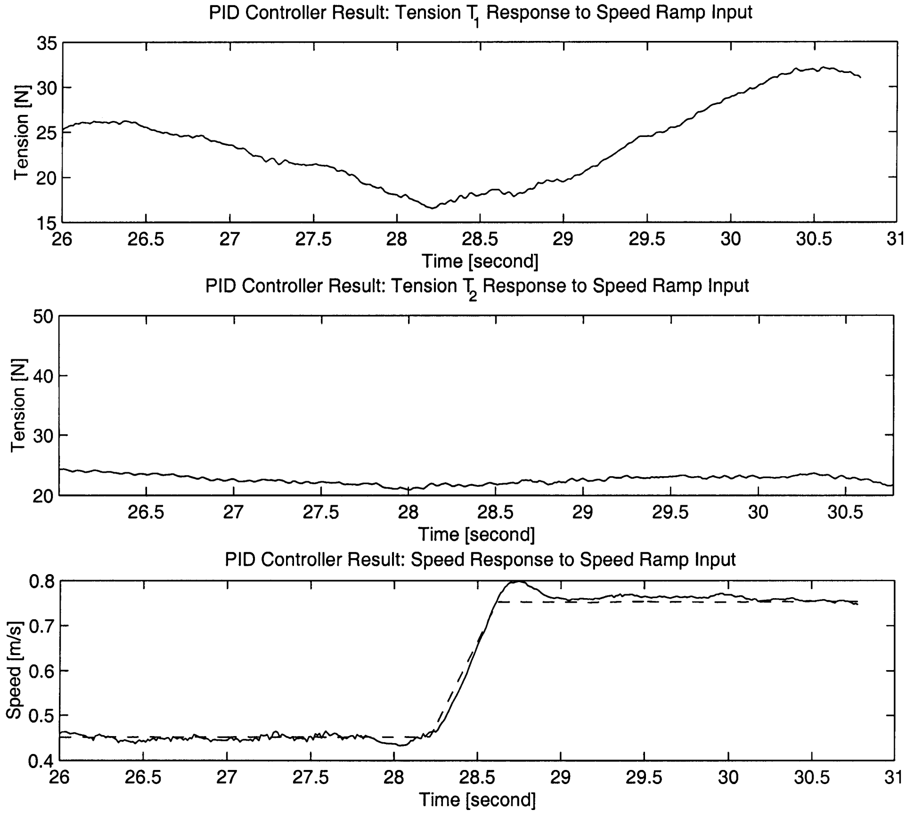


Fig. 11. Output response of Rotoflex web machine under a PID controller.

controller" is only marginally worse than the "perfect robust servomechanism controller," and yet it required far less effort to construct since no mathematical model of the web system was required to be obtained.

ACKNOWLEDGMENT

The authors are grateful to S.A. Bortoff and S. Krebb for helpful discussions held during this study.

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