MIMO Antenna Subset Selection With Space-Time Coding

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***Abstract*—This paper treats multiple-input multiple-output (MIMO) antenna subset selection employing space-time coding. We consider two cases differentiated based on the type of channel knowledge used in the selection process. We address both the selection algorithms and the performance analysis. We first consider the case when the antenna subsets are selected based on exact channel knowledge (ECK). Our results assume the transmission of orthogonal space-time block codes (with emphasis on the Alamouti code). Next, we treat the case of antenna subset selection when statistical channel knowledge (SCK) is employed by the selection algorithm. This analysis is applicable to general space-time coding schemes. When ECK is available, we show that the selection algorithm chooses the antenna set that maximizes the channel Frobenius norm leading to both coding and diversity gain. When SCK is available, the selection algorithm chooses the antenna set that maximizes the determinant of the covariance of the vectorized channel leading mostly to a coding gain. In case of ECK-based selection, we provide analytical expressions for average SNR and outage probability improvement. For the case when SCK-based selection is used, we derive expressions for coding gain. We also present extensive simulation studies, validating our results.**

***Index Terms*—Antenna subset selection, MIMO, space-time coding.**

I. Introduction

MIMO (multiple-input multiple-output) technology significantly enhances system performance. The extra degrees of freedom afforded by the multiple antennas can be used for increasing bit rates through spatial multiplexing [1]–[4] or for improved diversity order through space-time coding techniques [5], [6]. However, multiple antenna deployment requires multiple RF chains (consisting of amplifiers, analog to digital converters, etc.) that are typically very expensive. There is, therefore, considerable incentive for low-cost, low-complexity techniques with the benefits of multiple antennas. Optimal antenna subset selection is one such technique. A selection of antenna elements, which are typically much cheaper than RF chains, is made available at the transmitter and/or receiver. Transmission/reception is performed through the optimal subset.

Previous Work

Early work in antenna selection has concentrated on the multiple-input single-output (MISO) channel or the single-input multiple-output (SIMO) channel. This includes the hybrid selection/maximal ratio combining approach in [7]. Recently, there has been an explosion of interest [8]–[13] in application of antenna subset selection techniques to MIMO links. In [8], a criterion for selecting antenna subsets to maximize the channel capacity is presented. In [9], an upper bound on the capacity of a system with antenna subset selection is derived. In [10], it is shown that antenna selection techniques applied to low-rank channels can increase capacity. A fast selection algorithm based on "water-pouring" type ideas is presented in [11]. In [12], an antenna selection algorithm for minimizing error rate in spatial multiplexing systems with linear receivers is presented. In [13], exact expressions for average SNR increase through antenna selection with Alamouti code transmission are presented.

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Contributions and Organization of Paper

In this paper, we provide a comprehensive theory of antenna subset selection algorithms and performance analysis when space-time coding techniques are used over the MIMO link. We treat antenna subset selection based on exact channel knowledge (ECK) and statistical channel knowledge (SCK). Our main contributions may be categorized as follows.

1) Selection Algorithms: We present algorithms for ECK- and SCK-based selection with the antenna sets selected to minimize the probability of error. The algorithms are derived for the case of joint selection at the transmit and receive ends. We show that when ECK is available, the selection algorithm chooses the antenna subsets that maximize the channel Frobenius norm. This minimizes the instantaneous probability of error and maximizes the SNR. When SCK is available, the selection algorithm chooses the antenna subsets that maximize the determinant of the covariance of the vectorized channel. The selected subset need not be optimal per channel instance, but it minimizes the average probability of error (APE) over all possible channel realizations. We further show that for certain channel models, joint (transmit and receive) subset selection becomes decoupled, allowing the transmit antennas to be chosen independently of the receive antennas and vice versa.

2) Performance Analysis: We show through extensive analysis that for either case, antenna subset selection leads to an increase in effective SNR visible through a parallel shift in the average symbol error rate curves. We term this effect as coding gain. In addition, ECK-based selection leads to a marked improvement in diversity. To make the analysis for ECK-based selection tractable, we have assumed that selection capability is available at one end only and that the channels exhibit uncorrelated fading. For ECK-based selection, we derive analytical expressions for improvements in average SNR that serves as a

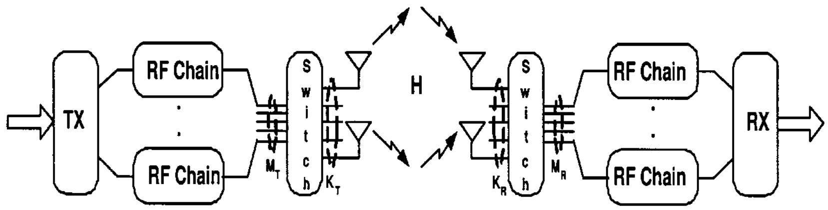

Fig. 1. System diagram, selection schematic.

pointer to coding gain. Next, we derive expressions for outage probability improvement that serves as an indicator of diversity gain. These results are restricted to the case of transmit selection with Alamouti code transmission or receive selection with two receive RF chains and any orthogonal space-time block code (OSTBC). The results hint that the diversity order obtained through antenna selection is the same as if all antennas were used. In the case of SCK-based selection, results are derived for the case of joint selection with correlated fading channels. We develop approximate expressions for coding gain. Diversity gain may also be visible under extreme channel conditions.

The paper is organized as follows. Section II presents the channel and signal model. Sections III and IV cover the main results for ECK and SCK based selection, respectively. We conclude with a summary of results in Section V.

II. Channel and Signal Model

Consider a point-to-point wireless link with M_T transmit and M_R receive RF chains. Assume that there are K_T ($K_T > M_T$) transmit and K_R ($K_R > M_R$) receive antenna elements. M_T out of K_T and M_R out of K_R antenna elements are selected and coupled to the transmit and receive RF chains, respectively; see Fig. 1. Transmission and reception over the MIMO channel (of size $M_R \times M_T$) is performed through these selected antenna subsets. We assume perfect channel state information and maximum likelihood decoding at the receiver. In addition, we also assume knowledge of channel statistics at the transmitter and receiver. Before proceeding further, we first describe the notation used in this paper for the reader's convenience. All vectors and matrices are in boldface.

$\mathbf{X}^T$	transpose operation.
$\mathbf{X}^H$	Hermitian transpose operation.
$\|\mathbf{X}\|_F$	Frobenius norm of $\mathbf{X}$.
$[\mathbf{X}]_{ij}$	(i, j)th element of $\mathbf{X}$.
$\mathbf{X} \otimes \mathbf{Y}$	Kronecker product of $\mathbf{X}$ with $\mathbf{Y}$.
$\mathcal{E}\{.\}$	expectation operator.
$\mathrm{vec}(\mathbf{X})$	vectorized matrix $\mathbf{X}$.

Channel Model: Let $\boldsymbol{\mathcal{H}}$ be the $(K_R \times K_T)$ channel matrix. The channel is assumed to be flat Rayleigh fading, remaining constant over a block of symbols and then changing independently to a new realization (quasistatic fading). While an i.i.d. channel assumption is common, in reality, measurements show the presence of transmit and receive correlation. Therefore, we assume correlated scattering at both the transmitter and receiver and that the channel matrix can be modeled as the product of a matrix inducing receive correlation, an i.i.d. complex Gaussian matrix, and a matrix inducing transmit correlation, i.e.,

$$\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{R}}_{\boldsymbol{R}}^{1/2} \boldsymbol{\mathcal{H}}_w \boldsymbol{\mathcal{R}}_{\boldsymbol{T}}^{1/2} \tag{1}$$

where $\boldsymbol{\mathcal{H}}_w$ is a $K_R \times K_T$ matrix with i.i.d. circular complex Gaussian elements with mean zero and variance one, i.e., $\mathcal{E}\{[\boldsymbol{\mathcal{H}}_w]_{ij}\} = 0$, $\mathcal{E}\{[\boldsymbol{\mathcal{H}}_w]_{ij}[\boldsymbol{\mathcal{H}}_w]_{ij}^H\} = 1$ and $\mathcal{E}\{[\boldsymbol{\mathcal{H}}_w]_{ij}[\boldsymbol{\mathcal{H}}_w]_{mn}^H\} = 0, \ \forall (i, j) \neq (m, n)$, $\boldsymbol{\mathcal{R}}_{\boldsymbol{T}}$, and $\boldsymbol{\mathcal{R}}_{\boldsymbol{R}}$ are the covariance matrices inducing transmit and receive correlation, respectively. We note that this model has been used extensively in [14] and [15].

Signal Model: Since there are only M_T transmit and M_R receive RF chains, we are constrained to transmit and receive from M_T out of K_T transmit and M_R out of K_R receive antennas, respectively. Henceforth, for ease of presentation, we denote the $(M_R \times M_T)$ channel between the selected antenna subsets by $\mathbf{H}$ and the transmit and receive covariance matrices for the selected channel as $\mathbf{R}_T$ and $\mathbf{R}_R$, respectively. Clearly, $\mathbf{H}$ is a subset of $\boldsymbol{\mathcal{H}}$, $\mathbf{R}_T$ is a principal sub-matrix of $\boldsymbol{\mathcal{R}}_{\boldsymbol{T}}$, and $\mathbf{R}_R$ is a principal sub-matrix of $\boldsymbol{\mathcal{R}}_{\boldsymbol{R}}$.

Let the symbols transmitted at the kth time instant from the M_T transmit antennas be $s_1[k] \cdots s_{M_T}[k]$. We have the following signal model:

$$\mathbf{y}[k] = \sqrt{\frac{E_S}{M_T}}\, \mathbf{H}\mathbf{s}[k] + \mathbf{n}[k] \tag{2}$$

where

$\mathbf{y}[k]\,(M_R \times 1)$	received signal vector;
E_S	total transmitted signal energy;
$\mathbf{s}[k] = [\, s_1[k] \ \ \cdots \ \ s_{M_T}[k] \,]^T$	transmitted signal vector at time k;
$\mathbf{n}[k]\,(M_R \times 1)$	additive white Gaussian noise vector with covariance $N_o \mathbf{I}_{M_R}$;
$\mathbf{H}$	$M_R \times M_T$ channel matrix between the selected transmit and receive antenna elements.

Assume that each frame is T symbol periods long. Then, if we stack the T received signal vectors

$$\mathbf{Y} = \sqrt{\frac{E_s}{M_T}}\, \mathbf{H}\mathbf{S} + \mathbf{N} \tag{3}$$

where $\mathbf{Y} = [\, \mathbf{y}_1 \ \ \cdots \ \ \mathbf{y}_T \,]$, $\mathbf{S} = [\, \mathbf{s}_1 \ \ \cdots \ \ \mathbf{s}_T \,]$, and $\mathbf{N} = [\, \mathbf{n}_1 \ \ \cdots \ \ \mathbf{n}_T \,]$. Using the channel model from (1), we have

$$\mathbf{Y} = \sqrt{\frac{E_s}{M_T}}\, \mathbf{R}_R^{1/2} \mathbf{H}_w \mathbf{R}_T^{1/2} \mathbf{S} + \mathbf{N}. \tag{4}$$

III. ECK-Based Selection

This section discusses ECK-based antenna subset selection. We focus on orthogonal space-time block coding and, in particular, on the Alamouti code. First, we present the algorithm for the case of joint antenna subset selection with correlated fading. The antennas are selected to minimize the instantaneous probability of error in the process also maximizing the received SNR of the data stream. This is followed by a detailed performance analysis. To make the analysis tractable, we make the assumptions of

1) selection capability at either the transmitter or receiver (not both together);
2) uncorrelated fading (i.e., $\boldsymbol{\mathcal{R}_T} = \mathbf{I}_{K_T}$ and $\boldsymbol{\mathcal{R}_R} = \mathbf{I}_{K_R}$).

We derive exact expressions for average SNR improvement. For the case of transmit selection with Alamouti code transmission, we also derive an expression for the improvement in outage probability.

A. OSTBC and Optimal Selection

We first motivate the use of the channel Frobenius norm as a selection metric when an OSTBC is transmitted over the channel. Then, we present the selection algorithm.

1) Review of OSTBC: Orthogonal space-time block codes provide maximal diversity order in a fading channel and have very low coding/decoding complexity. Coding and decoding is performed [16], [17] in such a way that the receive SNR of the data stream is

$$\gamma = \gamma_o \|\mathbf{H}\|_F^2 \tag{5}$$

where $\gamma_o = E_s/M_T N_o$. The probability of symbol error P_e is upper bounded by

$$P_e \leq e^{-d\gamma_o \|\mathbf{H}\|_F^2} \tag{6}$$

where d is a constant depending on the constellation. Full rate $r = 1$ OSTBCs for complex constellations exist only when the transmitter has two antennas. However, codes for more than two transmit antennas with complex constellations can be designed at the expense of rate [17], e.g., the maximum known code rate for $M_T = 3$ and $M_T = 4$ is $r = 3/4$.

From (5) and (6), we may conclude that maximizing the channel Frobenius norm maximizes SNR as well as minimizes the instantaneous probability of error. We use this observation to develop the selection algorithm.

2) Antenna Selection Algorithm: There are a total of $\binom{\mathbf{K}_T}{\mathbf{M}_T}\binom{\mathbf{K}_R}{\mathbf{M}_R}$ possible selections of transmit and receive antennas. The optimal antenna subset is chosen to minimize the instantaneous probability of error. It is clear from the observations made above that the optimal antenna subsets should maximize the channel Frobenius norm. For joint selection at the transmitter and receiver, this corresponds to finding the $(M_R \times M_T)$ submatrix of the $(K_R \times K_T)$ channel with the highest Frobenius norm. In case of transmit side selection $(K_R = M_R)$ or receive side selection $(K_T = M_T)$ side only, the algorithm selects the M_T out of K_T columns or M_R out of K_R rows with the highest Frobenius norm, respectively. Joint selection requires computation of the Frobenius norms of a total of $\binom{\mathbf{K}_T}{\mathbf{M}_T}\binom{\mathbf{K}_R}{\mathbf{M}_R}$ possible selections and then a search procedure for the maximum norm. This computation should not be a problem in practical systems where the number of transmit and receive antenna elements rarely exceeds four to five.

B. Performance Analysis

ECK-based selection leads to significant performance improvement in the form of coding gain as well as increased diversity order. We develop analysis for average SNR that serves as a pointer to coding gain. This analysis is general and is applicable to any OSTBC. Next, we derive an expression for outage probability improvement that indicates diversity gain. These results are restricted to the cases of transmit selection with Alamouti code transmission (or receive selection with $M_R = 2$ and any OSTBC). We present extensive simulations supporting our analysis.

1) Average SNR Analysis: We develop analysis for transmit selection $K_R = M_R$ in this section (easily extended to receive selection). The selection algorithm chooses the M_T out of K_T transmit antenna elements that maximize the Frobenius norm of the channel. The SNR of the received data stream (5) is

$$\gamma = \gamma_o \|\mathbf{H}\|_F^2. \tag{7}$$

Let $T_k,\ k = 1 \cdots K_T$ be the Frobenius norms of the K_T columns of the channel matrix ($\boldsymbol{\mathcal{H}}$). $T_k^2,\ k = 1 \cdots K_T$ are i.i.d. chi-squared variables with probability density function (p.d.f.) given by

$$f(z) = f(T_k^2 = z) = \frac{z^{M_R - 1} e^{-z}}{(M_R - 1)!} \tag{8}$$

and cumulative distribution function (c.d.f.) given by

$$F(z) = p(T_k^2 \leq z) = 1 - \sum_{l=0}^{M_R - 1} e^{-z} \frac{z^l}{l!}. \tag{9}$$

The average SNR $\mathcal{E}\{\gamma\}$ is

$$\mathcal{E}\{\gamma\} = \gamma_o \mathcal{E}\{\|\mathbf{H}\|_F^2\} = \gamma_o \sum_{i=1}^{M_T} \mathcal{E}\{T_{n_i}^2\} \tag{10}$$

where $n_i,\ i = 1 \cdots M_T$ are the indices of the M_T selected transmit antennas.

Equation (10) and the fact that the selection algorithm chooses the columns with the highest Frobenius norm suggest that we are interested in the first-order statistics of the M_T highest variables among K_T i.i.d. chi-squared variables with p.d.f. and c.d.f., per (8) and (9).

For purposes of analysis, we generate a new set of ordered variables $X_k,\ k = 1 \cdots K_T$ from $T_k^2,\ k = 1 \cdots K_T$ such that $X_{K_T} \geq \cdots \geq X_k \geq \cdots \geq X_1$. X_{K_T} is the largest of K_T random variables distributed according to (8). X_k is the kth largest of K_T random variables, and so on. The average SNR $\mathcal{E}\{\gamma\}$ is

$$\mathcal{E}\{\gamma\} = \gamma_o \mathcal{E}\{X_{K_T}\} + \cdots + \gamma_o \mathcal{E}\{X_{K_T - M_T + 1}\}. \tag{11}$$

TABLE I
EXPECTED VALUE OF HIGHEST AND SECOND HIGHEST ORDERED STATISTICS, TRANSMIT SELECTION, ALAMOUTI CODE

	$K_T = 3$	$K_T = 4$	$K_T = 5$	$K_T = 6$
$M_R = 2$	3.213	3.547	3.808	4.02
	1.824	2.210	2.502	2.738
$M_R = 3$	4.495	4.891	5.197	5.446
	2.821	3.308	3.667	3.953
$M_R = 4$	5.731	6.177	6.520	6.798
	3.819	4.390	4.805	5.132

We are now ready to state our first result. The average value of the X_kth statistic is

$$\mathcal{E}\{X_k\} = \frac{K_T!}{(k-1)!(K_T-k)!(M_R-1)!} \sum_{r=0}^{k-1} (-1)^r \cdot \binom{k-1}{r} \sum_{s=0}^{(M_R-1)\zeta} a_s(M_R, \zeta) \frac{\Gamma(1+M_R+s)}{(\zeta+1)^{(1+M_R+s)}} \quad (12)$$

where $\zeta = K_T - k + r$ and $a_s(M_R, \zeta)$ is the coefficient of x^s in the expansion of $(\sum_{l=0}^{M_R-1}(x^l/l!))^\zeta$. Details of the derivation are provided in the Appendix.

Equation (12) has been tabulated in [18] for $1 \le M_R \le 4$ and $1 \le K_T \le 40$. These values can be substituted in (11) to obtain the average SNR. We reproduce some values for $M_T = 2$ (Alamouti code) in Table I. The two rows (in the table) for each M_R correspond to $\mathcal{E}\{X_{K_T}\}$ and $\mathcal{E}\{X_{K_T-1}\}$, respectively. We define the gain g in average SNR as

$$g = \log_{10}\left(\frac{\mathcal{E}\{\gamma\}}{\gamma_o M_R M_T}\right) = \log_{10}\left(\frac{\mathcal{E}\left\{\sum_{i=1\cdots M_T} T_{n_i}^2\right\}}{M_R M_T}\right) \quad (13)$$

where $\gamma_o M_R M_T$ is the average SNR for the OSTBC with no antenna selection (or random antenna selection). The selection gain g can be substantial, as we will see shortly.

Finally, note that the analysis for receive antenna selection $K_T = M_T$, $K_R > M_R$ is easily obtained by replacing K_T with K_R and M_R with M_T in (12).

• *Simulations*: Each curve in Fig. 2 depicts the gain g in average SNR for transmit antenna selection ($K_T \ge M_T$, $K_R = M_R$) with $M_T = 2$. We note significant improvement in average SNR. In addition, observe that the gain with transmit antenna selection is higher when fewer number of receive antennas are used. This makes sense since, as the number of receive antennas increases, the column squared Frobenius norms, which are essentially chi-squared variables, look increasingly equal, thereby reducing selection leverage. The curves in Fig. 3 depict the average SNR gain for receive antenna selection $K_R \ge M_R$ with Alamouti code transmission ($M_T = 2$). The gains with receive antenna selection for the case $M_R = 2$ are the same as the gains with transmit antenna selection with two receive antennas (top-most curves in Figs. 2 and 3). This is as it should be since these two cases are identical from an analytical point of view.

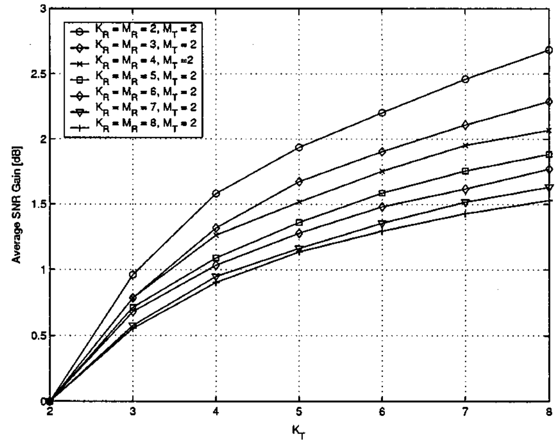

Fig. 2. Average SNR gain with ECK-based transmit selection (Alamouti code).

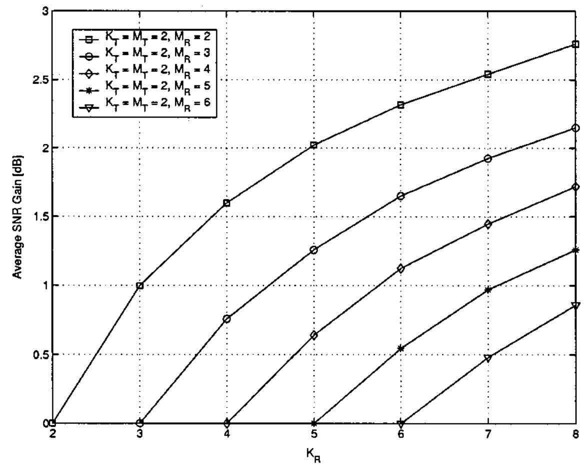

Fig. 3. Average SNR gain with ECK-based receive selection (Alamouti scheme).

2) Outage Probability—Transmit Selection With Alamouti Code: Antenna selection results in increased system diversity with the improvement in outage capacity/probability being a good indicator of such an increase. In this section, we derive an approximate expression for outage probability improvement that is revealed by simulations to be quite accurate. We restrict our treatment to transmit antenna selection with $M_T = 2$ (Alamouti code transmission). The results allow us to conclude that the diversity order achievable through antenna selection is the same as if all antennas were in use.

The outage capacity [4] C_{out} corresponding to an outage probability p_{out} is defined as

$$P(C \le C_{out}) = p_{out} \quad (14)$$

i.e., p_{out} is the probability that the capacity C is less than a certain outage value C_{out}. The capacity (maximum achievable mutual information) of the system ($M_T = 2$, Alamouti code transmission) with transmit antenna selection is clearly given by

$$C = \log_2\left(1 + \gamma_o(X_{K_T} + X_{K_T-1})\right). \quad (15)$$

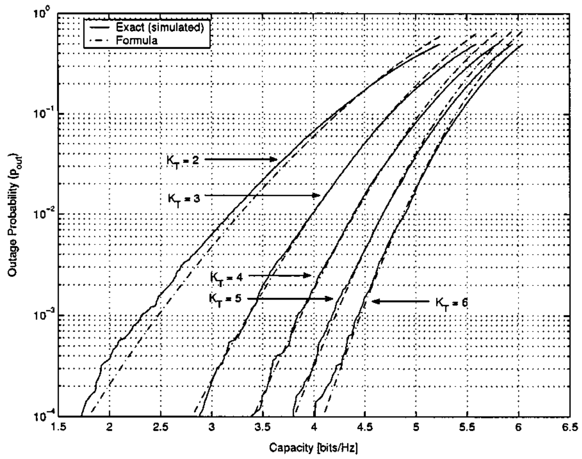

Fig. 4. Outage probability with ECK-based transmit selection (exact and approximated) $M_R = 2$.

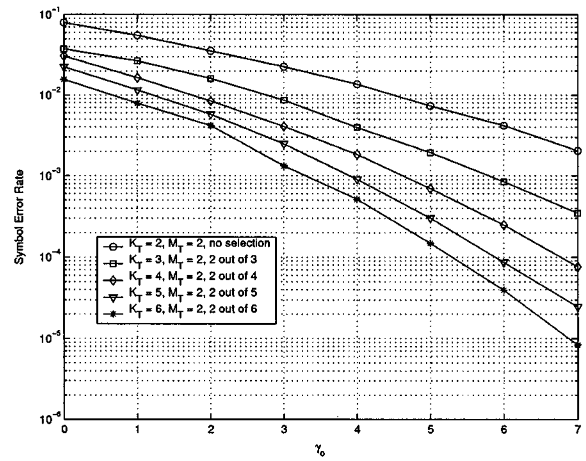

Fig. 5. SER curves for ECK-based transmit selection (Alamouti code, 2 Rx antennas, 4-QAM).

X_{K_T} and X_{K_T-1} are as defined previously. We have

$$P(C \le C_{out}) = P\left((X_{K_T} + X_{K_T-1}) \le \frac{2^{C_{out}} - 1}{\gamma_o} = k\right) = \int_0^{k/2}\int_0^x f_{XY}(x, y)\,dy\,dx + \int_{k/2}^{k}\int_0^{k-x} f_{XY}(x, y)\,dy\,dx \quad (16)$$

where we have replaced X_{K_T} and X_{K_T-1} with X and Y, respectively, for notational convenience. f_{XY} is the joint distribution of the highest and second highest order statistic and is given by

$$f_{XY} = K_T(K_T - 1)F(y)^{K_T-2}f(x)f(y) \quad (17)$$

where $F(.)$ and $f(.)$ are as defined in (8) and (9), respectively. Equation (16) works out to

$$P(C \le C_{out}) = F\left(\frac{k}{2}\right)^{K_T} + K_T\int_0^{k/2} f(k-x)F(x)^{K_T-1}\,dx. \quad (18)$$

The second expression is tedious to integrate. Instead, we simplify the computation by working with an approximation. We have

$$P(C \le C_{out}) \approx F\left(\frac{k}{2}\right)^{K_T} + K_T\int_0^{k/2} f(x)F(x)^{K_T-1}\,dx = 2 * F\left(\frac{k}{2}\right)^{K_T}. \quad (19)$$

• *Simulations*: The approximated theoretical outage probability is compared to the exact value in Fig. 4. The figure reveals (19) to be a very tight bound in the region of interest (i.e., in the outage region). The improvement in outage probability is considerable. For example, the outage probability for $C_{out} = 4.2$ bits/Hz reduces drastically from 10% with no selection ($M_R = K_R = 2$) to about 2.0% with $K_T = 3$ to about 0.5% for $K_T = 4$. Note that the outage behavior of received SNR is easily obtained from the above analysis.

Diversity Order: Equation (19) indicates that the slope of the outage probability curve is the same as if all K_T antennas were used. Furthermore, note the following inequality:

$$\sum_{i=1}^{K_T} X_i \ge X_{K_T} + X_{K_T-1} \ge \frac{2}{K_T}\sum_{i=1}^{K_T} X_i \quad (20)$$

where the first inequality is obvious, and the second inequality follows from the fact that the sum of the first N variables of an ordered set (of positive variables) is greater than N times the average of the set. Since $\sum_{i=1}^{K_T} X_i = \|\mathcal{H}\|_F^2$, (20) may be rewritten as

$$\|\mathcal{H}\|_F^2 \ge X_{K_T} + X_{K_T-1} \ge \frac{2}{K_T}\|\mathcal{H}\|_F^2. \quad (21)$$

The parameter of interest, *viz.* $X_{K_T} + X_{K_T-1}$, is bounded between two quantities both of which are chi-squared with $2K_TM_R$ degrees of freedom. This hints at K_TM_R order diversity.[1] From (19) and (21), we may conclude that transmit antennas selection extracts the same diversity gain as if all K_T antennas were used.

Figs. 5 and 6 depict the average symbol error rate curves for transmit and receive antenna selection, respectively, for various selection configurations. We assume the transmission of the Alamouti code. Both coding and diversity gain promised by the analysis are visible through the parallel shift (coding gain) and the increased slope (increased diversity order) of the curves. Increase in K_T or K_R increases system diversity as well as coding gain. Furthermore, observe that the curves (in Fig. 6) corresponding to $K_R = 3$, $M_R = 2$ and $K_R = 3$, $M_R = 3$ have the same slope, i.e., selecting two out of three antennas provides the same diversity as if all three antennas were in use. This substantiates the observation made above that antenna subset selection extracts the same diversity order as if all antennas were used.

This completes the discussion on ECK-based selection. In the following section, we focus on selection algorithms and performance analysis based on channel statistics.

[1]This inequality can be easily extended to cover the case of any orthogonal space-time code and for receive selection as well.

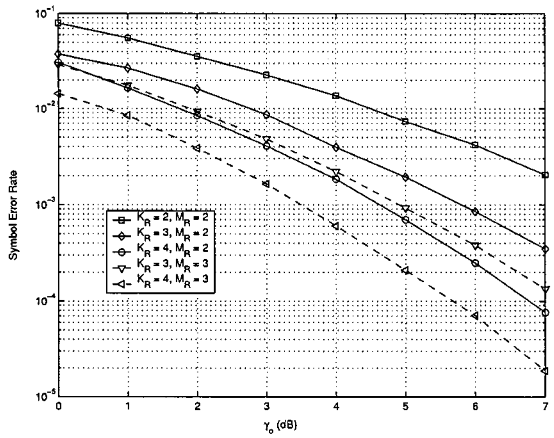

Fig. 6. SER curves for ECK based receive selection (Alamouti code, 2 Tx antennas, 4-QAM).

IV. SCK-Based Selection

ECK-based antenna selection may not always be feasible, especially when the channel changes rapidly. In such scenarios, we can attempt subset selection based on channel statistics. Channel statistics generally depend on large-scale scatterers/distance between the transmitter and receiver [19], [20] and change slowly, if at all, even in mobile environments. This section focuses on SCK-based antenna subset selection for general space-time codes. Unlike ECK, in which the selected antenna sets minimize the *instantaneous* probability of error, when SCK is available, we target our selection toward minimizing the *average* probability of error. Towards this goal, we first develop a general expression for the APE in correlated fading. This expression assumes that the antennas sets have been selected at the transmitter and receiver and remain the same over all channel instances. We use this expression to motivate antenna subset selection and to develop the selection algorithm. We further show that joint selection becomes decoupled and that SCK-based selection leads mostly to a coding gain. Diversity gain might also be available under extreme channel conditions.

A. APE and Antenna Selection

In this section, we derive expressions for the APE in correlated fading channels. We use the special form of the expression to develop an antenna selection algorithm.

1) APE: Let $\mathbf{S}^{(i)}$ be the transmitted codeword, and let $\mathbf{S}^{(j)}$ be some other codeword that is not equal to the transmitted codeword. Define the (i, j)th error matrix as $\mathbf{E}_{i,j} = \mathbf{S}^{(i)} - \mathbf{S}^{(j)}$. The probability of decoding the codeword $\mathbf{S}^{(j)}$ instead of $\mathbf{S}^{(i)}$, where $j \neq i$, is given by the pairwise error probability (PEP)

$$\mathrm{P_e}\left(\mathbf{S}^{(i)} \rightarrow \mathbf{S}^{(j)} \middle| \mathbf{H}\right) \leq e^{-(E_s/4M_T N_o) D_{i,j}} \tag{22}$$

where we have used the Chernoff bound, and $D_{i,j} = \|\mathbf{H}(\mathbf{S}^{(i)} - \mathbf{S}^{(j)})\|_F^2$. It is easily verified that $D_{i,j}$ may be expressed as

$$D_{i,j} = \boldsymbol{\Omega}\boldsymbol{\mathcal{E}}_{i,j}\boldsymbol{\mathcal{E}}_{i,j}^H\boldsymbol{\Omega}^H \tag{23}$$

where $\boldsymbol{\Omega} = \mathrm{vec}(\mathbf{H}^T)^T$ and $\boldsymbol{\mathcal{E}}_{i,j} = \mathbf{I}_{M_R} \otimes \mathbf{E}_{i,j}$. From (22) and (23), the PEP averaged over the channel statistics is

$$\begin{aligned}\mathrm{P_e}(\mathbf{S}^i \rightarrow \mathbf{S}^j) &\leq \mathcal{E}\left\{e^{-\gamma\|\boldsymbol{\Omega}\boldsymbol{\mathcal{E}}_{i,j}\|_F^2}\right\} \\ &= \frac{1}{\det\left(\mathbf{I}_{M_R M_T} + \gamma\mathbf{R}\boldsymbol{\mathcal{E}}_{i,j}\boldsymbol{\mathcal{E}}_{i,j}^H\right)}\end{aligned} \tag{24}$$

where $\gamma = E_S/4M_T N_o$, and $\mathbf{R} = \mathcal{E}\{\boldsymbol{\Omega}^H\boldsymbol{\Omega}\}$. When the channel is as per (1), we have $\mathbf{R} = \mathbf{R}_R \otimes \mathbf{R}_T$. We reiterate that $\mathbf{R}_T$ and $\mathbf{R}_R$ are principal sub-matrices of $\boldsymbol{\mathcal{R}_T}$ and $\boldsymbol{\mathcal{R}_R}$, respectively, and correspond to one particular selection. Furthermore, note that

$$\begin{aligned}\mathbf{R}\boldsymbol{\mathcal{E}}_{i,j}\boldsymbol{\mathcal{E}}_{i,j}^H &= (\mathbf{R}_R \otimes \mathbf{R}_T)(\mathbf{I}_{M_R} \otimes \mathbf{E}_{i,j})(\mathbf{I}_{M_R} \otimes \mathbf{E}_{i,j}^H) \\ &= \mathbf{R}_R \otimes \mathbf{R}_T\mathbf{E}_{i,j}\mathbf{E}_{i,j}^H\end{aligned} \tag{25}$$

so that we may rewrite (24) as

$$\mathrm{P_e}\left(\mathbf{S}^{(i)} \rightarrow \mathbf{S}^{(j)}\right) \leq \frac{1}{\det\left(\mathbf{I}_{M_T M_R} + \gamma\mathbf{R}_R \otimes (\mathbf{R}_T\mathbf{E}_{i,j}\mathbf{E}_{i,j}^H)\right)}. \tag{26}$$

At high SNR, this reduces to

$$P_e\left(\mathbf{S}^{(i)} \rightarrow \mathbf{S}^{(j)}\right) \leq \frac{\gamma^{-(r_{\mathbf{R}_R} r_{\mathbf{R}_T})}}{\left(\prod_{m=1}^{r_{\mathbf{R}_R}} \sigma_m\right)^{r_{\mathbf{R}_T}} \left(\prod_{n=1}^{r_{\mathbf{R}_T}} \lambda_n\right)^{r_{\mathbf{R}_R}}} \tag{27}$$

where $r_{\mathbf{R}_R}$ and $r_{\mathbf{R}_T}$ are the ranks of $\mathbf{R}_R$ and $\mathbf{R}_T$, respectively, σ_m is the mth singular value of $\mathbf{R}_R$, and λ_n is the nth singular value of $\mathbf{R}_T\mathbf{E}_{i,j}\mathbf{E}_{i,j}^H$. If $\mathbf{R}_R$ and $\mathbf{R}_T$ are both full rank, then we have

$$P_e\left(\mathbf{S}^{(i)} \rightarrow \mathbf{S}^{(j)}\right) \leq \frac{\gamma^{-(M_T M_R)}}{\det(\mathbf{R}_R)^{M_T}\det(\mathbf{R}_T)^{M_R}\det(\mathbf{E}_{i,j}\mathbf{E}_{i,j}^H)^{M_R}}. \tag{28}$$

Equation (27) shows the dependence of the diversity order on the rank of the transmit and receive covariance matrices. Specifically, the diversity order is driven by the product of the rank of $\mathbf{R}_T$ and $\mathbf{R}_R$. A decrease in rank by one in $\mathbf{R}_T$ causes the diversity order to decrease by the rank of $\mathbf{R}_R$, and vice versa. We also note a coding gain hit due to the transmit and receive correlations.

The results in (27) and (28) correspond to one particular selection of transmit and receive antenna subsets. In general, the transmit and receive covariance matrices are different for different selections, and the goal is to choose the antenna sets that minimize (28).

2) Antenna Selection Algorithm: We now present the algorithm for joint antenna subset selection with correlated fading. The choice made by the algorithm need not be optimal per channel instance, i.e., it may not minimize the instantaneous probability of error, rather it minimizes the APE over all possible channel realizations. There are a total of $\binom{K_T}{M_T}\binom{K_R}{M_R}$ possible selections. However, we can leverage the result in (28) to note that *under the assumptions of the channel model (1), the antenna subset selection at the transmitter and receiver becomes decoupled*. Joint selection may be performed by selecting the optimal transmit antennas independently of the receiver, and vice versa. Transmit selection is performed by choosing the principal sub-matrix $\mathbf{R}_T$ of $\boldsymbol{\mathcal{R}_T}$ with the highest determinant. Receive selection is performed by choosing the principal sub-matrix $\mathbf{R}_R$ of $\boldsymbol{\mathcal{R}_R}$ with the highest determinant. Note that because of this decoupling, we need to search over only $\binom{K_T}{M_T} + \binom{K_R}{M_R}$ selections instead of $\binom{K_T}{M_T}\binom{K_R}{M_R}$, as before.

B. Performance Analysis

We define the selection gain as the performance improvement through transmission on the optimal antenna sets instead of any other selection. Let $\mathbf{R}_{T_{opt}}$ and $\mathbf{R}_{R_{opt}}$ be the transmit and receive covariance, respectively, corresponding to the optimal antenna sets. Let $\mathbf{R}_T$ and $\mathbf{R}_R$ be the corresponding covariance matrices for any other selection. From (28), we may conclude that selection gain is mostly manifested in the form of a coding gain. In extreme situations, we may also have diversity gain.

1) Coding Gain: Assume that $\mathbf{R}_{opt}$ and $\mathbf{R}$ are both full rank. From (28), the performance improvement is given by

$$\left(\frac{\det(\mathbf{R}_T)}{\det(\mathbf{R}_{T_{opt}})}\right)^{M_R}\left(\frac{\det(\mathbf{R}_R)}{\det(\mathbf{R}_{R_{opt}})}\right)^{M_T}.$$

The effect, at high SNRs, is the same as if there were coding in the system (i.e., a parallel shift in the symbol error rate curves). Hence, we term this gain as "coding gain," which is approximately given by

$$\frac{10}{M_T}\log_{10}\left(\frac{\det(\mathbf{R}_{T_{opt}})}{\det(\mathbf{R}_T)}\right) + \frac{10}{M_R}\log_{10}\left(\frac{\det(\mathbf{R}_{R_{opt}})}{\det(\mathbf{R}_R)}\right).$$

2) Diversity Gain: In extremely correlated channel conditions, it is possible that some subsets of the antennas will be fully correlated. In these cases, either the transmit or receive covariance matrices (or both) may be low rank. Since the rank of the covariance matrix plays a role in the diversity advantage, statistical selection may also improve the diversity advantage (by increasing the slope of the error rate curve). Let r and t be the rank of $\mathbf{R}_R$ and $\mathbf{R}_T$, respectively. Then, the diversity order gain is given by $r_{opt}t_{opt} - rt$, where r_{opt} and t_{opt} are the ranks of $\mathbf{R}_{R_{opt}}$ and $\mathbf{R}_{T_{opt}}$, respectively.

A diversity effect (region of SNR where the curves corresponding to optimal selection and any other selection show variable slopes) may also be visible in lower SNR regimes when both covariance matrices are full rank with ill-conditioned $\mathbf{R}$ and well-conditioned $\mathbf{R}_{opt}$. For example, the curves for different antenna sets have different slopes for certain regions of SNR, indicating difference in diversity order even if the corresponding covariance matrices have the same rank. To understand this effect, consider the following simplified case.

Assume the transmission of an OSTBC and that there is no transmit correlation, i.e., $\boldsymbol{\mathcal{R}_T} = \mathbf{I}_{K_T}$. Assume also that $\mathbf{R}_R$ is ill-conditioned and that $\mathbf{R}_{R_{opt}}$ is well-conditioned. For an OSTBC, $\mathbf{E}_{i,j}\mathbf{E}_{i,j}^H = \delta\mathbf{I}_{M_T}$, where δ is a constant. Inserting $\mathbf{R}_T = \mathbf{I}_{M_T}$ in (26)

$$\begin{aligned} P_e&\left(\mathbf{S}^{(i)} \to \mathbf{S}^{(j)}\right) \\ &\le \frac{1}{\det(\mathbf{I}_{M_R} + \gamma\delta\mathbf{R}_R)^{M_T}} = \left(\prod_{k=1}^{M_R}(1+\gamma\delta\sigma_k)\right)^{-M_T} \\ &= \left(1 + \sum_{k=1}^{M_R}\gamma^k\delta^k w_k(\sigma_1, \ldots, \sigma_{M_R})\right)^{-M_T} \end{aligned} \tag{29}$$

where σ_k, $k=1\cdots M_R$ is the kth singular value of $\mathbf{R}_R$ and $w_k(\sigma_1, \ldots, \sigma_{M_R})$ is a function of the singular values, e.g., $w_1(\sigma_1, \ldots, \sigma_{M_R}) = \sum_{k=1}^{M_R}\sigma_k$ and $w_{M_R}(\sigma_1, \ldots, \sigma_{M_R}) = \prod_{k=1}^{M_R}\sigma_k$.

Assume that $\mathbf{R}_R$ is full rank but ill-conditioned with $\sigma_{M_R} \ll \sigma_k, k = 1\cdots M_R - 1$. Then, for a certain range of γ, we have $\gamma\sigma_{M_R} \ll 1$, implying $\rho^{M_R-1}w_{M_R-1}(\sigma_1, \ldots, \sigma_{M_R}) \gg \rho^{M_R}w_{M_R}(\sigma_1, \ldots, \sigma_{M_R})$, which in turn implies that the slope of the curve (diversity order) is $M_T(M_R - 1)$. However, at high enough SNR, we have $\gamma\sigma_{M_R} \gg 1$, implying $\rho^{M_R-1}w_{M_R-1}(\sigma_1, \ldots, \sigma_{M_R}) \ll \rho^{M_R}w_{M_R}(\sigma_1, \ldots, \sigma_{M_R})$ so that we have full M_TM_R-order diversity. If $\mathbf{R}_{R_{opt}}$ is well-conditioned, there are regions of SNR where the corresponding symbol error rate curve has full M_TM_R-order diversity, whereas the curve for $\mathbf{R}_R$ exhibits $M_T(M_R - 1)$-order diversity.

In general, we can conclude that if we have a well-conditioned $\mathbf{R}_{opt}$ and an ill-conditioned $\mathbf{R}_R$, we will observe a difference in diversity order in lower ranges of SNR. At high enough SNR, however, the curves will eventually become parallel (with the same diversity order).

• *Simulations*: We assume correlated fading with the channel model per (1). We use the covariance matrices in (30) generated using the GWSSUS model for correlated fading. See [19] for more details on the model. Fig. 7 depicts the average symbol error rate curves for two scenarios with receive selection ($K_R = 3$, $M_R = 2$, $K_T = M_T = 2$), where the transmit covariance has been assumed to be identity, and the receive covariance matrices for the two scenarios are given by $\boldsymbol{\mathcal{R}_{R_I}}$ and $\boldsymbol{\mathcal{R}_{R_{II}}}$, respectively. Fig. 8 depicts the average symbol error rate curves for a system with transmit and receive selection ($K_R = 3$, $M_R = 2$, $K_T = 3$, $M_T = 2$), where the transmit covariance matrix is given by $\boldsymbol{\mathcal{R}_{R_I}}$, and the receive covariance matrix is given by $\boldsymbol{\mathcal{R}_{R_{II}}}$

$$\begin{aligned} \boldsymbol{\mathcal{R}_{R_I}} &= \begin{bmatrix} 1.00 & 0.2e^{0.86\pi j} & 0.8e^{-0.26\pi j} \\ 0.2e^{-0.86\pi j} & 1.00 & 0.2e^{0.86\pi j} \\ 0.8e^{0.26\pi j} & 0.2e^{-0.86\pi j} & 1.0000 \end{bmatrix} \\ \boldsymbol{\mathcal{R}_{R_{II}}} &= \begin{bmatrix} 1.00 & 0.91e^{j\pi/2} & -0.69 \\ 0.91e^{-j\pi/2} & 1.00 & 0.91e^{j\pi/2} \\ -0.69 & 0.91e^{-j\pi/2} & 1.00 \end{bmatrix}. \end{aligned} \tag{30}$$

• *Discussion*:

—The selection algorithm (based on maximizing determinant of the covariance; see Table II) predicts that the best receive antennas are 1 and 2 (or 2 and 3) for scenario I and receive

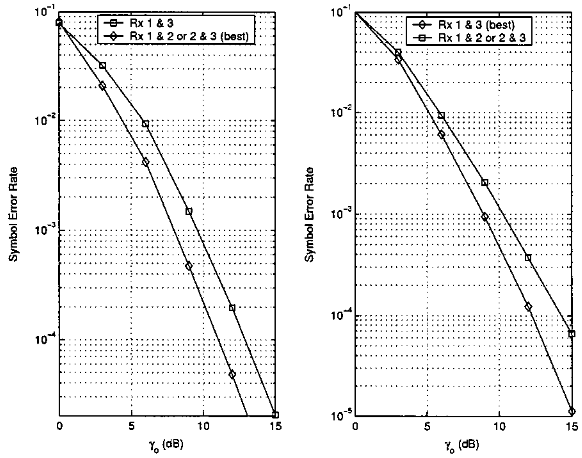

Fig. 7. SER curves for SCK based receive selection (Alamouti code, two Rx antennas, 4-QAM, scenarios I and II).

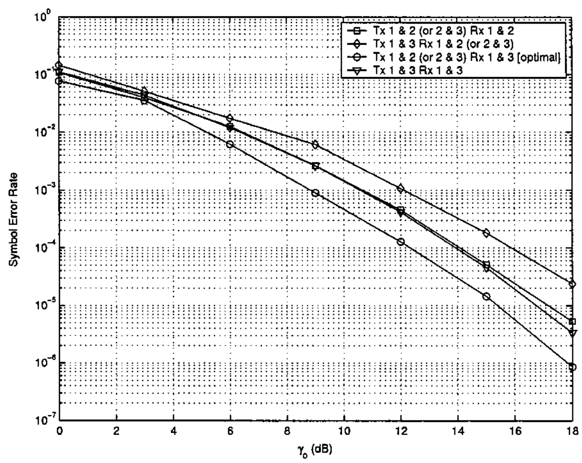

Fig. 8. SER curves for joint SCK-based selection (Alamouti code, 4-QAM).

TABLE II
SELECTION RULE FOR SIMULATED SCENARIOS

	1 & 2 or (2 & 3)	1 & 3
Scenario I $\det(\mathbf{R}_R)$	0.96 √	0.35
Scenario II $\det(\mathbf{R}_R)$	0.17	0.52 √

antennas 1 and 3 for scenario II. The prediction is accurate as the corresponding curves in Fig. 7 are clearly optimal in terms of having the minimum average symbol error rate. Further, note the diversity effect in scenario II, even though the covariance matrices corresponding to both selections are full rank. This is because the covariance matrix for receive antennas 1 and 2 is very badly conditioned (but still of rank 2), whereas the covariance matrix for antennas 1 and 3 is better conditioned.

—Fig. 8 depicts the four possible choices for joint transmit and receive selection. The algorithm predicts that transmit antennas 1 and 2 and receive antennas 1 and 3 are optimal and is clearly validated in the figure.

—Clearly, statistical selection can significantly enhance performance (2–4 dB in the examples considered). In general, these gains will vary depending on the exact structure of $\boldsymbol{\mathcal{R}_T}$ and $\boldsymbol{\mathcal{R}_R}$.

V. CONCLUSIONS AND COMMENTS

Since the cost of transmit/receive RF chains is a significant factor limiting the use of multiple antennas in wireless links, we believe that antenna subset selection, which is a low-cost low-complexity technique that retains the diversity benefits of multiple antennas, is a promising solution. In this paper, we addressed the problem of optimal MIMO subset selection with space-time coding in flat fading channels. We developed algorithms and performance analysis for ECK- and SCK-based selection. We characterized the selection gain and presented extensive simulations corroborating our analysis.

The analysis for ECK-based selection assumed i.i.d. channels for analytical tractability. In reality, the channels will be correlated, and the selection gains, especially the diversity benefit, will be mitigated. The exact performance loss depends on the degree of correlation present in the channel. Furthermore, we have limited our treatment in this paper to flat fading channels. The selection gains in presence of delay spread might be mitigated due to sufficient frequency diversity in the channel. This is an interesting topic for future work.

APPENDIX
DERIVATION OF AVERAGE SNR THROUGH ORDERED STATISTICS APPROACH

From (11), the average SNR is

$$\mathcal{E}\{\gamma\} = \gamma_0 \mathcal{E}\{X_{K_T}\} + \cdots + \gamma_o \mathcal{E}\{X_{K_T - M_T + 1}\} \tag{31}$$

where X_i, $i = 1 \cdots K_T$ has been defined earlier. Essentially, we are interested in the first-order moments of the M_T highest ordered statistics of K_T i.i.d. chi-squared variables. The p.d.f. of the kth highest statistic is

$$p(x) = \frac{K_T!}{(k-1)!(K_T-k)!} F(x)^{k-1}(1-F(x))^{K_T-k} f(x) \tag{32}$$

where

$$f(x) = \frac{x^{M_R - 1} e^{-x}}{(M_R - 1)!}$$
$$F(x) = 1 - \sum_{j=0}^{M_R - 1} \frac{x^j e^{-x}}{j!}. \tag{33}$$

We have

$$\begin{aligned}\mathcal{E}\{X_i\} &= \frac{K_T!}{(i-1)!(K_T-i)!} \\ &\quad \cdot \int_0^\infty x F(x)^{i-1}(1-F(x))^{K_T-i} f(x)\, dx \\ &= \frac{K_T!}{(i-1)!(K_T-i)!} \sum_{r=0}^{i-1} (-1)^r \binom{i-1}{r} \\ &\quad \cdot \int_0^\infty x(1-F(x))^{K_T-i+r} f(x)\, dx\end{aligned}$$

$$
\begin{aligned}
&= C \sum_{r=0}^{i-1} (-1)^r \binom{i-1}{r} \int_0^{\infty} e^{-(K_T - i + r + 1)} x^{M_R} \\
&\quad \cdot \left(\sum_{l=0}^{M_R - 1} \frac{x^l}{l!} \right)^{K_T - i + r} dx \\
&= C \sum_{r=0}^{i-1} (-1)^r \binom{i-1}{r} \sum_{s=0}^{(M_R - 1)(K_T - i + r)} \\
&\quad \cdot a_s(M_R, K_T - i + r) \\
&\quad \cdot \frac{M_R + s}{(K_T - i + r + 1)^{M_R + s + 1}}
\end{aligned} \tag{34}
$$

where

$$C = \frac{K_T!}{(i-1)!(K_T - i)!(M_R - 1)!}$$

and $a_s(M_R, K_T - i + r)$ is the coefficient of x^s in the expansion of

$$\left(\sum_{l=0}^{M_R - 1} \frac{x^l}{l!} \right)^{K_T - i + r}.$$

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