Sliding Mode Trained Neural Control for Single and Coupled Inverted Pendulum System

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Abstract

This study applies a sliding-mode-based neural network to control inverted pendulum systems. Neural network weights are updated using a cost function which denotes distance from the sliding manifold. Thus, minimizing the cost function equals reaching the sliding surface. Sliding mode based neural network also makes the system robust to uncertainties in parameters and dynamical uncertainties. Chattering effect is solved by modifying the cost function. Simulations are fulfilled for a SISO and a MIMO model of an inverted pendulum. The results of simulations reveal the effectiveness of proposed method.

Keywords

Cost Function; Inverted Pendulum; Neural Control; Sliding Mode

Introduction

Due to hardship in obtaining control efforts in some applications, adaptive controllers like neural networks have been widely used [Bose (2007)]. In a neural controller, weights which construct the control effort are updated in order to minimize a cost function. One way to combine good features of classical control theory with neural networks is deriving a cost function which yields classical control goals. Cost function can be depicted as distance from a sliding manifold. This way, minimizing cost function is equal to approaching desired state values.

One drawback of sliding mode control is the chattering effect. The chattering is generally undesirable because it involves extremely high control activities and may excite high-frequency dynamics neglected in modelling [Slotine & Li (1991)].

In [Yildiz et al. (2007)], the signal control obtained from ADALINE neural network is updated using a cost function which denotes distance from the sliding manifold. Thus, merge good features of sliding mode and neural network. The method is applied to physical model of the linear servo drive.

In [Wang et al. (2002)], a supervisory controller is appended into the FNN controller to force the states to be within the constraint set. Therefore, if the FNN controller cannot maintain the stability, the supervisory controller starts working to guarantee stability. On the other hand, if the FNN controller works well, the supervisory controller will be deactivated. The method is applied to an inverted pendulum system. We use this system for our simulations.

Recently [Kayacan et al. (2013)], the control of a spherical rolling robot by using an adaptive neuro-fuzzy controller in combination with a sliding-mode control (SMC)-theory-based learning algorithm has been presented. The proposed control structure consists of a neuro-fuzzy network and a conventional controller which is used to guarantee the asymptotic stability of the system in a compact space.

In this study the sliding-mode-based neural controller is applied to SISO and MIMO model of an inverted pendulum. The cost function is derived from Lyapunov stability criteria. As the cost function becomes smaller, outputs tend to the desired values and weights updating decreases. Simulations are brought afterwards.

This paper is organized as follows: Problem statements given in Section 2. Section 3 presents the structure of neural network. Simulation examples to demonstrate the performance of the proposed method is provided in Section 4. Section 5 gives the conclusions of the advocated design methodology.

Problem Statement

Model of the System

In this paper dynamics of a system including *m* subsystems is considered. Dynamics are described by $y_i^{n_i-1} = h_i + b_i u_i + g_i$, where $y_i^{l_i}$ is the lth derivative of y_i considering $x = [y_1, \dot{y}_1, \ldots, y_1^{n_i-1}, \ldots, y_m, \dot{y}_m, \ldots, y_m^{n_i-1}]^T$ as state vector. The subsystems can be described as follows:

$$\dot{x} = f(x) + B(x) + d \tag{1}$$

In which $x^T \in \Re^n$ is the state vector, $n = \sum_{i=1}^{m} n_i$, $u \in \Re^m$ is the control vector and $f(x) \in \Re^n$ is an unknown, bounded and continuous function. $B(x) \in \Re^{nxm}$ is the input matrix, with continuous and bounded elements and $rank(B(x))|_{\forall x} = m$. $d \in \Re^n$ describes output disturbance, assumed bounded. All elements of $f(x) \in \Re^n$ and $d \in \Re^n$ are bounded. Fully-actuated mechanical systems can be described in the form of equation (1).

Control Design

Control law is derived from SMC structure. First, an appropriate sliding mode is selected to ensure dynamics' convergence to desired values. Control signal should be derived such that Lyapunov conditions are satisfied. Selecting the Lyapunov function using sliding mode is a natural and reasonable approach to get to the desired control goals that is tracking desired trajectory.

Sliding mode

For system described in equation (1), one choice for the sliding mode is

$$\sigma = Ge_t = 0 \tag{2}$$

The tracking error vector is defined as $e_t = \left[e_1, \ldots, e_1^{(n_1-1)}, \ldots, e_m, \ldots, e_m^{(n_m-1)}\right]^T \in \Re^n$, in which $e_i = y_{d_i} - y_i$, $\sigma = [\sigma_1, \ldots, \sigma_m]^T \in \Re^m$ and $G \in \Re^{m \times n}$. Matrix G has to be Hurwitz, to damp tracking error and its derivatives. Thus each elements of the vector $\sigma(e)$ is a function of output error. $\sigma_i = \sum_{j=1}^{n_i-1} a_{ij} e_i^{(j)}$ with $a_{ij} > 0, a_{i1} = 1$ describes function of tracking error and its derivatives, which has some roots in left half of s plane.

Deriving Control Signal

One Lyapunov function candidate is:

$$V = \tfrac{1}{2}\sigma^T \sigma \tag{3}$$

where $V \in \Re$. We can also assume $V = (1/2)\|\sigma\|_2^2$ in which $\|.\|_2$ reveals Euclidean norm with initial condition $V(0) = 0$. Time derivative of the Lyapunov function has to be negative definite to ensure stability. We can equate $\dot{V}$ to a negative definite function, as following:

$$\dot{V} = -\sigma^T D\sigma - \mu \frac{\sigma^T \sigma}{\|\sigma\|_2} \tag{4}$$

D is a symmetric positive definite matrix and $\mu > 0$. Replacing (3) in (4), we have

$$\sigma^t \left(\dot{\sigma} + D\sigma + \mu \frac{\sigma}{\|\sigma\|_2}\right) = 0 \tag{5}$$

For $\sigma \neq 0$, the control law is derived from:

$$\left(\dot{\sigma} + D\sigma + \mu \frac{\sigma}{\|\sigma\|_2}\right) = 0 \tag{6}$$

Thus sliding mode conditions are satisfied. Discontinuous term should be small enough in order to prevent the chattering effect in SMC. Because simulations are performed in discrete form, we can neglect the discontinuous term. Thus control signal is selected such that $(\dot{\sigma} + D\sigma) = 0$. For further analysis, $(D\sigma)$ can be replaced by $(D\sigma + \mu\sigma/\sigma^T\sigma)$.

For a system in the form of (1) and a sliding surface as (2), a control signal satisfying $(\dot{\sigma} + D\sigma) = 0$ is:

$$\boldsymbol{u} = -(GB)^{-1}\left(G(f + d - \dot{x}_{d_i}) - D\sigma\right) = u_{eq} + (GB)^{-1}D\sigma \tag{7}$$

Where $x_d = \left[y_{d_1}, \ldots, y_{d_1}^{(n_1-1)}, \ldots, y_{d_m}, \ldots, y_{d_m}^{(n_m-1)}\right]^T$, u_{eq} is called equivalent control and is derived from $\dot{\sigma} = 0$.

In most of the works in literature which are combined Neural Networks with SMC, u_{eq} is derived from a neural network [Kaynak et al. (2001), Morioka et al. (1995), Jezernik et al. (1997)]. Though, here we use a one-layer MLP network to get the whole control signal.

Neural Network Structure

One Layer MLP

The structure used for the neural network in this study is shown in fig. 1.

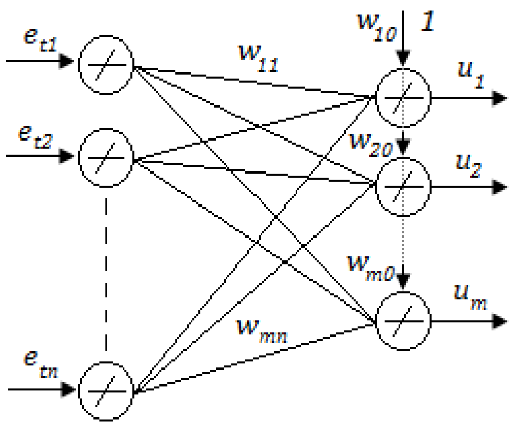

FIG. 1 NEURAL NETWORK STRUCTURE

e_{t_i} is the ith row of e_t vector, w_{ij} shows weight between the ith and the jth nodes and w_{i0} denotes bias term. Control inputs are defined as:

$$u_i = \sum_{j=1}^{n} e_{t_i} w_{ij} + 1w_{i0}, \qquad i = 1, \dots, m. \quad (8)$$

In fig. 1 activation functions are linear and the neural network is static. Equation (8) denotes a PD controller for a second-order system. For higher order systems, it denotes a state feedback controller. From (8), when tracking errors are zero, control signal equals the bias term, which rejects disturbance effect.

In this paper, weights update is selected such that$(\dot{\sigma} + D\sigma) = 0$. Thus Lyapunov conditions are satisfied. The cost function used is as follows:

$$\boldsymbol{E} = \frac{1}{2}(\dot{\sigma} + D\sigma)^T(\dot{\sigma} + D\sigma) \quad (9)$$

As $E \to 0$ with weights update, $(\dot{\sigma} + D\sigma) = 0$ is satisfied, the states move on the sliding surface and converge to the desired values.

Control law is derived from SMC structure. First, an appropriate sliding mode is selected to ensure dynamics' convergence to desired values. Control signal should be derived such that Lyapunov conditions are satisfied. Selecting the Lyapunov function using sliding mode is a natural and reasonable approach to get to the desired control goals which is tracking desired trajectory.

Updating Weights

Weights are updated as following:

$$\dot{w}_{ij} = -\eta \frac{\partial E}{\partial w_{ij}} \quad (10)$$

$\eta > 0$ is the learning coefficient. Using Chain rule, we have

$$\dot{w}_{ij} = -\eta \frac{\partial E}{\partial u_i}\frac{\partial u_i}{\partial w_{ij}} \quad (11)$$

$$\dot{w}_{ij} = -\eta(\dot{\sigma} + D\sigma)^T \frac{\partial \dot{\sigma}}{\partial u_i} e_{t_j} \quad (12)$$

$$\dot{w}_{ij} = -\eta(\dot{\sigma} + D\sigma)^T \frac{\partial (G\dot{x}_d - G\dot{x})}{\partial u_i} e_{t_j} \quad (13)$$

If we rewrite equation (1) as

$$\dot{x} = f(x) + [B_1(x) \vdots \cdots \vdots B_m(x)] \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix} + d \quad (14)$$

Replacing (14) in (13) concludes

$$\dot{w}_{ij} = -\eta(\dot{\sigma} + D\sigma)^T G B_i(x) e_{t_j} \quad (15)$$

For updating bias weights, w_{i0}, we have

$$\dot{w}_{i0} = \boldsymbol{\eta}(\dot{\sigma} + D\sigma)^T G B_i(x) \quad (16)$$

If we select nonlinear activation functions instead of linear ones, updating weights is similar to the above procedure, though updating terms are multiplied by the derivative of the activation function, that is $\dot{w}_{ij} = \eta(\dot{\sigma} + D\sigma)^T \dot{g}_i G B_i(x) e_{t_j}$.

In the above approach when the cost function equals zero, that is $(\dot{\sigma} + D\sigma) = 0$, updating stops and the states reach the desired values. In [Yildiz et al. (2007)], It is shown that the minimum of cost function is global, because the second derivative of cost function always stays positive.

Stability

The Lyapunov candidate is selected as

$$\boldsymbol{V} = \frac{1}{2}(\dot{\sigma} + D\sigma)^T(\dot{\sigma} + D\sigma) \quad (17)$$

It can be easily shown that$V > 0$, while$(\dot{\sigma} + D\sigma) \neq 0$. When$(\dot{\sigma} + D\sigma) = 0$, we have$V = 0$. Differentiate the above equation, we have

$$\dot{V} = -\sum_{i=1}^{m}\sum_{j=0}^{n} \frac{\partial V}{\partial w_{ij}} \frac{dw_{ij}}{dt} + g(\gamma)\dot{\gamma} \quad (18)$$

$g(\gamma)$ is the derivative of V on other parameters. Replacing (10) in (24), we have

$$\dot{V} = -\sum_{i=1}^{m}\sum_{j=0}^{n} \left(\frac{\partial V}{\partial w_{ij}}\right)^2 + g(\gamma)\dot{\gamma} \quad (19)$$

To ensure stability, learning rate should be chosen large enough. Thus, derivative of Lyapunov function will be negative definite, $\dot{V} < 0$.

Simulations

SISO Case

Here we will apply sliding-mode-based neural controller for two cases. First we consider SISO case with an inverted pendulum system that is shown in fig. 2 [Wang et al. (2002)].

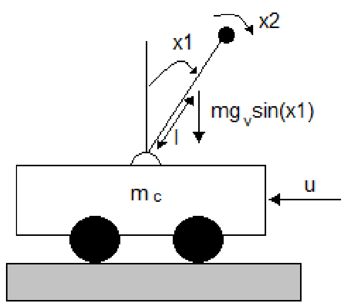

FIG 2 THE INVERTED PENDULUM SYSTEM

If we choose$x_1 = \theta$to be the angle of the pendulum with respect to the vertical line, the dynamic equations of the inverted pendulum system are

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (f + gu + d)$$
$$\boldsymbol{y} = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (20)$$

Where

$$\boldsymbol{f} = \frac{g_v \sin x_1 - (mlx_2^2 \cos x_1 \sin x_1)}{l\left(\frac{4}{3} - \frac{m \cos^2 x_1}{m_c + m}\right)}$$
$$\boldsymbol{g} = \frac{\frac{\cos x_1}{m_c+m}}{l\left(\frac{4}{3} - \frac{m\cos^2 x_1}{m_c+m}\right)} > 0 \tag{21}$$

$g_v = 9.8\, meter/(\sec^2)$ is the acceleration due to gravity, m_c is the mass of the cart, $\boldsymbol{l}$ is the half-length of the pole, m is the mass of the pole and u is the control input. Here, we assume $m_c = 1kg$, $m = 0.1kg$ and $l = 0.5$ meter. For implementing sliding-mode-based neural controller on this continuous system, we should discretize it with a proper sampling time. Thus, updating states would be as following:

$$\boldsymbol{X}(k+1) = X(k) + Ts \times dX(k+1) \tag{22}$$

Structure of system with controller is shown in fig. 3. Desired trajectory for angle of pendulum is a sinusoid. We assume that the neural network is a one-layer linear network. To control the angle of pendulum, the sliding manifold is chosen as $\sigma = \dot{e} + Ce$, where $e = \theta_r - \theta$ refers to the error in angle of pendulum. We select controller parameters as $C = 2$, $D = 2$ and $\eta = 0.1$, and the sampling time $T_s = 0.001s$.

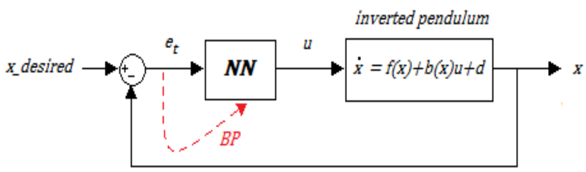

FIG. 3 STRUCTURE OF SYSTEM WITH CONTROLLER

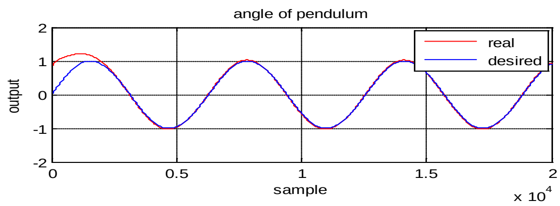

FIG 4 TRACKING ANGLE OF PENDULUM

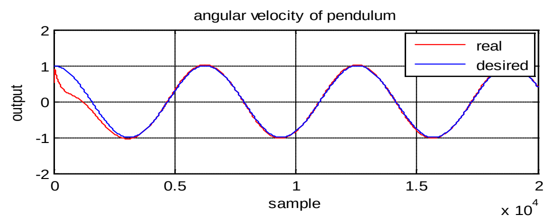

FIG 5 TRACKING ANGULAR VELOCITY OF PENDULUM

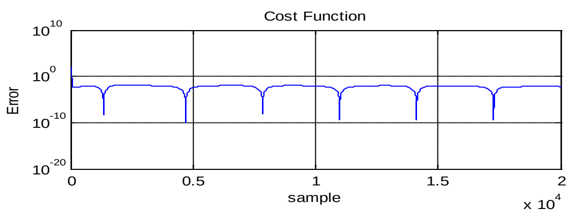

FIG 6 TRACKING ERROR OF PENDULUM

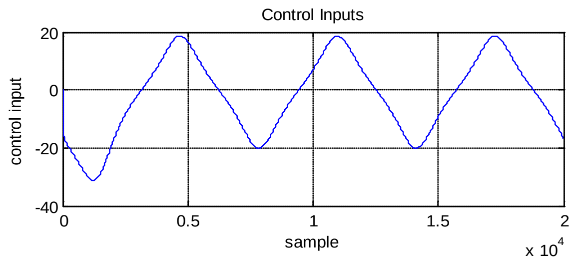

FIG 7 CONTROL INPUT APPLIED TO CART

Figs. 4-7 show the response of the system to the sinusoid reference input for angle. Fig. 4 shows the output trajectory θ and reference output θ_r, where the reference trajectory is tracked perfectly and error is hardly noticeable. Fig. 5 shows the second state of the system, which is angular velocity of pendulum and tracked perfectly similar to angle of pendulum. Fig. 6 shows the tracking error in logarithmic axis. It can be seen, error decreases fast during tracking and remains in a limit bound for steady-state. In fig. 7 the control signal is shown, that is seen to be smooth.

The presented results show that sliding-mode neural controller works suitably, and the states converge to sliding surface properly. This convergence is achieved by a simple weight update algorithm and an uncertain system with limited knowledge on system parameters.

MIMO Case

In MIMO case, we consider two inverted pendulums connected by a moving spring mounted on two carts (fig. 8) [Yang et al. (2010)]. In this system position of the pivot in moving spring is a function of time, which can change along the full length l of the pendulums. The inputs of the system are torque u_i applied at the pivot point of each pendulum. The motion of carts is assumed to be sinusoid trajectories. Each pendulum is assumed as a decoupled subsystem of the whole system. The objective is to control the angle of each pendulum with only its information so that each pendulum tracks its own desired reference trajectory while the connected spring and carts are moving.

If we define $\boldsymbol{x}_i = [\theta_i, \dot{\theta}_i]^T = [x_{i1}, x_{i2}]^T, i = 1,2,$ as the systems states, the dynamical equations of the coupled pendulums can be described as following:

$$\dot{\boldsymbol{x}}_1 = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \boldsymbol{x}_1 + \begin{bmatrix} 0 \\ \frac{1}{cml^2} \end{bmatrix} u_1 + \begin{bmatrix} 0 & 0 \\ \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \boldsymbol{x}_2 - \left[\frac{m}{M} \sin(x_{11})\, x_{12}^2 + \frac{k(a(t)-cl)}{cml^2} (y_1 - y_2) \right] \tag{23}$$

$$\dot{\boldsymbol{x}}_2 = \begin{bmatrix} 0 & 1 \\ \frac{g}{cl} - \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \boldsymbol{x}_2 + \begin{bmatrix} 0 \\ \frac{1}{cml^2} \end{bmatrix} u_2 + \begin{bmatrix} 0 & 0 \\ \frac{ka(t)(a(t)-cl)}{cml^2} & 0 \end{bmatrix} \boldsymbol{x}_1 - \begin{bmatrix} \frac{m}{M}\sin(x_{21})\, x_{22}^2 + \frac{k(a(t)-cl)}{cml^2}(y_2 - y_1) \end{bmatrix} \qquad (24)$$

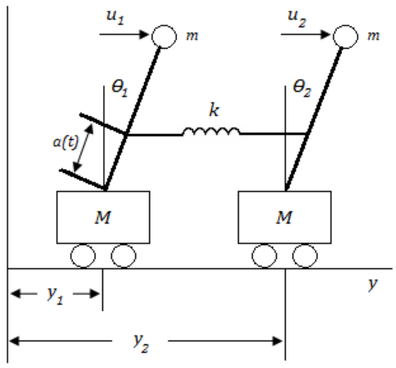

FIG 8 THE COUPLED DOUBLE INVERTED PENDULUM

Where $C = M/(M+m)$, k and g are spring and gravity constants and u_1 and u_2 are pendulums control torques. We choose $g = 9.8$, $l = 1$, $k = 1$, $M = m = 4$ for simulations. The motions of the carts are assumed to be sinusoids, that is $y_1 = \sin(\omega_1 t)$ and $y_2 = L + \sin(\omega_2 t)$, where L is the natural length of the spring and $\omega_1 \neq \omega_2$. Here, we select $\omega_1 = 2$, $L = 2$ and$\omega_2 = 3$. Also, we choose $a(t) = \sin 5t$. Considering $X = [\boldsymbol{x}_1, \boldsymbol{x}_2]^T = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$ as the whole system states, we can reach the standard form of equation (1).

Again we consider sine-wave trajectories as desired angle of pendulums. We also assume one-layer linear network for sliding-mode-based neural controller. The sliding manifold for the first subsystem is chosen as $\sigma_1 = \dot{e}_1 + Ce_1$, where $e_1 = \theta_{1_r} - \theta_1$ refers to the error in angle of first pendulum. For the second subsystem the sliding manifold is chosen as $\sigma_2 = \dot{e}_2 + Ce_2$, where $e_2 = \theta_{2_r} - \theta_2$ refers to the error in angle of the second pendulum. We select controller parameters as $C = 2$, $D = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$, which is positive definite,$\eta = 0.4$, and the sampling time $T_s = 0.001s$.Both references are applied at the same time.

Figs. 9-14 show the response of the system to the sine-wave reference for each angle. As shown in Figs. 9-10, the output trajectory θ_1 tracks the reference output θ_{1_r} perfectly, while simultaneously output trajectory θ_2 tracks the reference outputθ_{2_r}. Figs. 11-12 show the second states of the subsystem, angular velocity of each pendulum, which are tracked perfectly similar to the angle of pendulums. Fig. 13 shows the tracking error in logarithmic space, which remains bounded in steady-state. As it is seen in fig. 14 the control signals are bounded and well-behaved.

The presented results show that sliding-mode-based neural controller works suitably for MIMO systems, where decoupled and interaction terms are assumed disturbance. All the states converge to sliding surface properly, and also the system is capable of coping with harmonic changes in references.

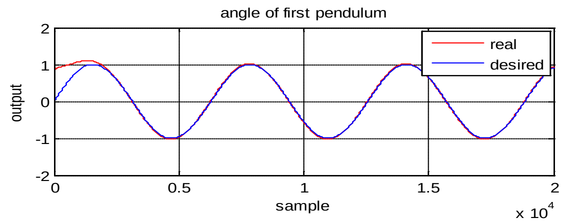

FIG 9 TRACKING ANGLE OF FIRST PENDULUM

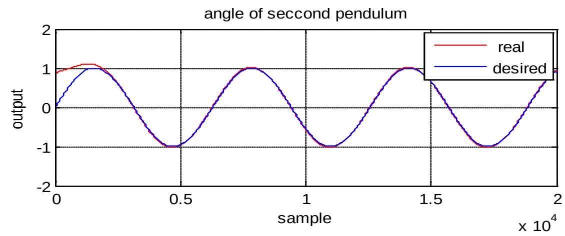

FIG 10 TRACKING ANGLE OF SECOND PENDULUM

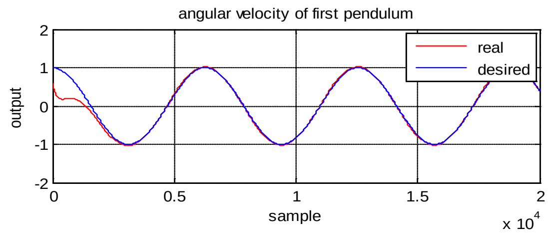

FIG 11 TRACKING ANGULAR VELOCITY OF FIRST PENDULUM

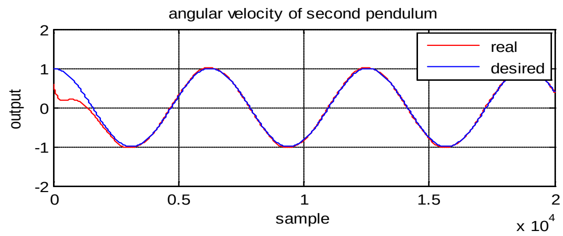

FIG 12 TRACKING ANGULAR VELOCITY OF SECOND PENDULUM

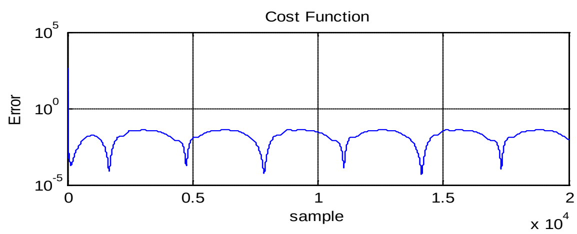

FIG 13 TRACKING ERRORM OF PENDULUMS

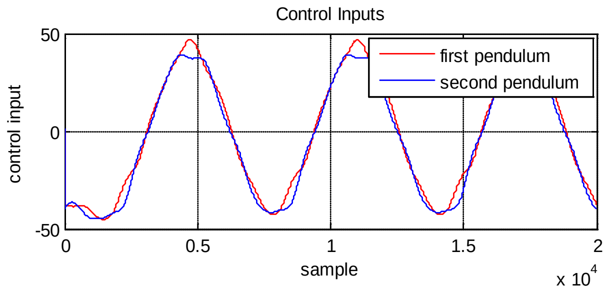

FIG 14 CONTROL INPUTS APPLIED TO EACH PENDULUM

Conclusion

In this paper a neural network based on sliding mode is proposed for an inverted pendulum. Weight adaptation in the neural network uses a cost function derived from Lyapunov stability criteria. The aim in this study is to develop a learning method for parameters of neural controller that not only can be applied without the need for calculating Jacobean of plant but also guarantees stability and robustness of the learning approach. According to the simulations for SISO and MIMO case, good tracking characteristics in outputs are obtained.

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